

Speed-aware network design

A parametric optimisation approach

Ugo Rosolia, Marc Bataillou Almagro, George Iosifidis, Amit Kumar, Georgios Paschos

Outline

- ▶ Motivation
- ▶ Problem formulation
- ▶ Solution strategy
- ▶ Results

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Middle Mile Network Design

Origin #1



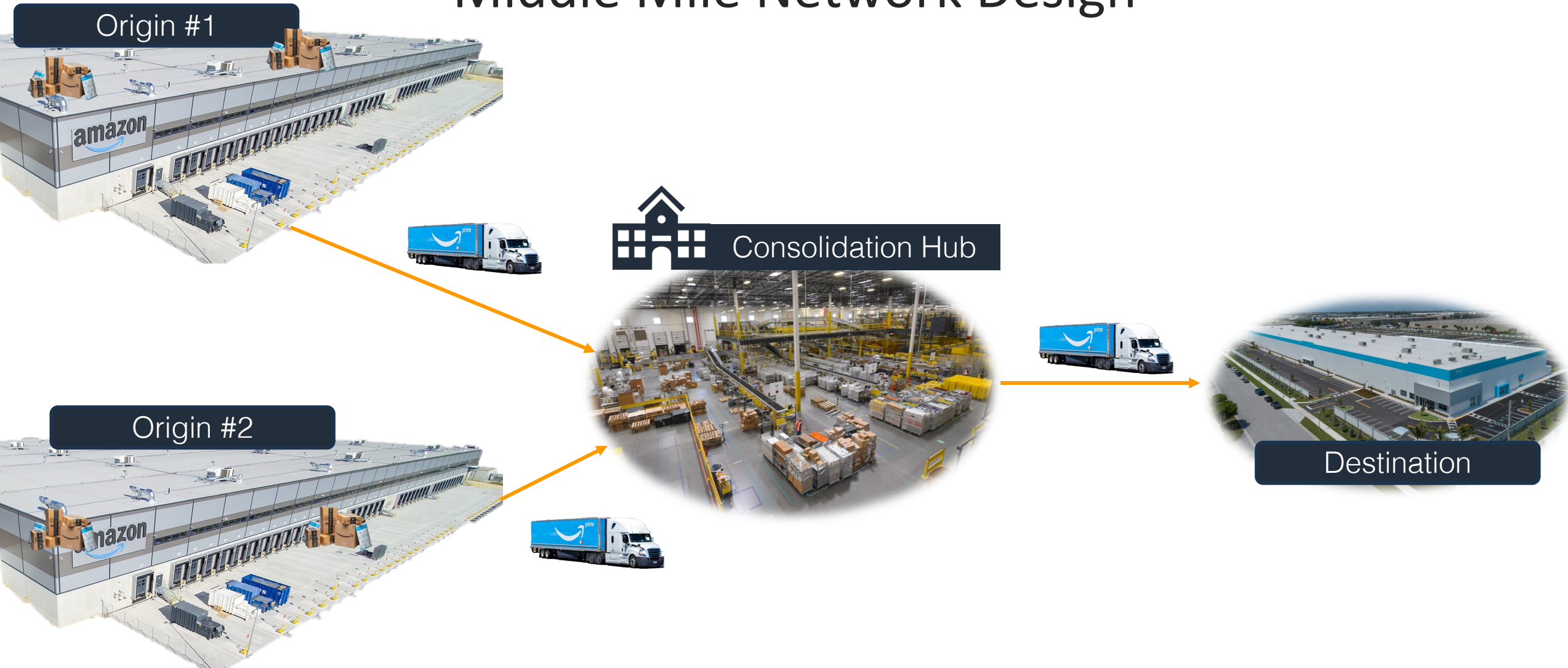
Origin #2



Destination



Middle Mile Network Design



Middle Mile Network Design



Middle Mile Network Design



Key decisions

1. Connectivity, i.e., buildings to connect.
2. Timing, i.e., trucks departure times.

Middle Mile Network Design



Key decisions

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2. Timing, i.e., trucks departure times.

Objectives

1. Reduce cost.
2. Minimize carbon emissions.
3. Maximize delivery speed.

Middle Mile Network Design



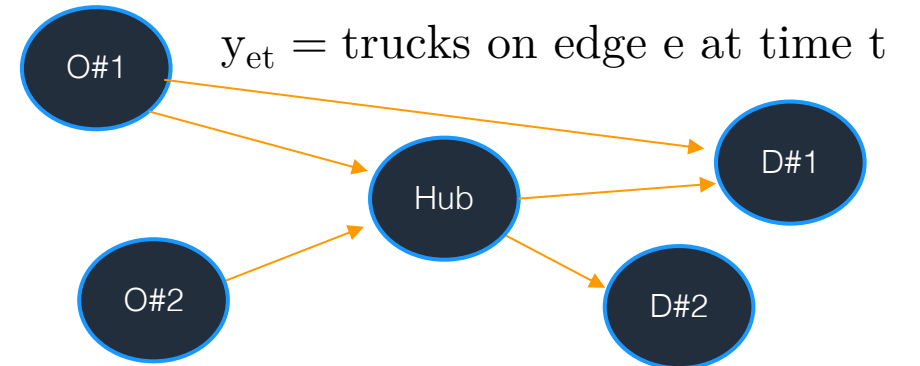
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Time-expanded network flow problem



Middle Mile Network Design



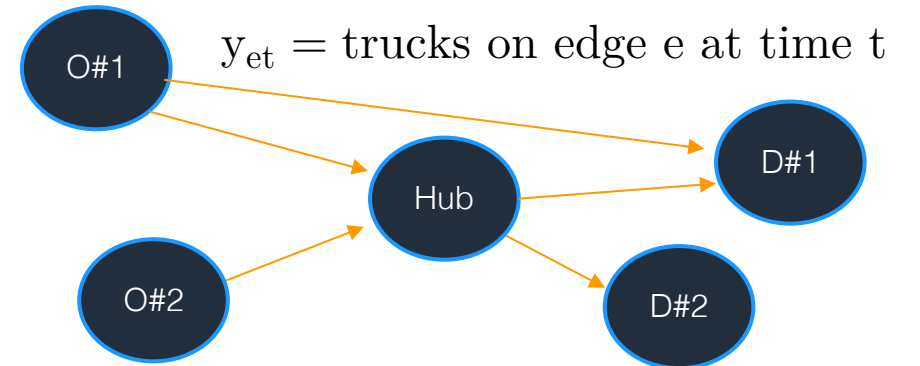
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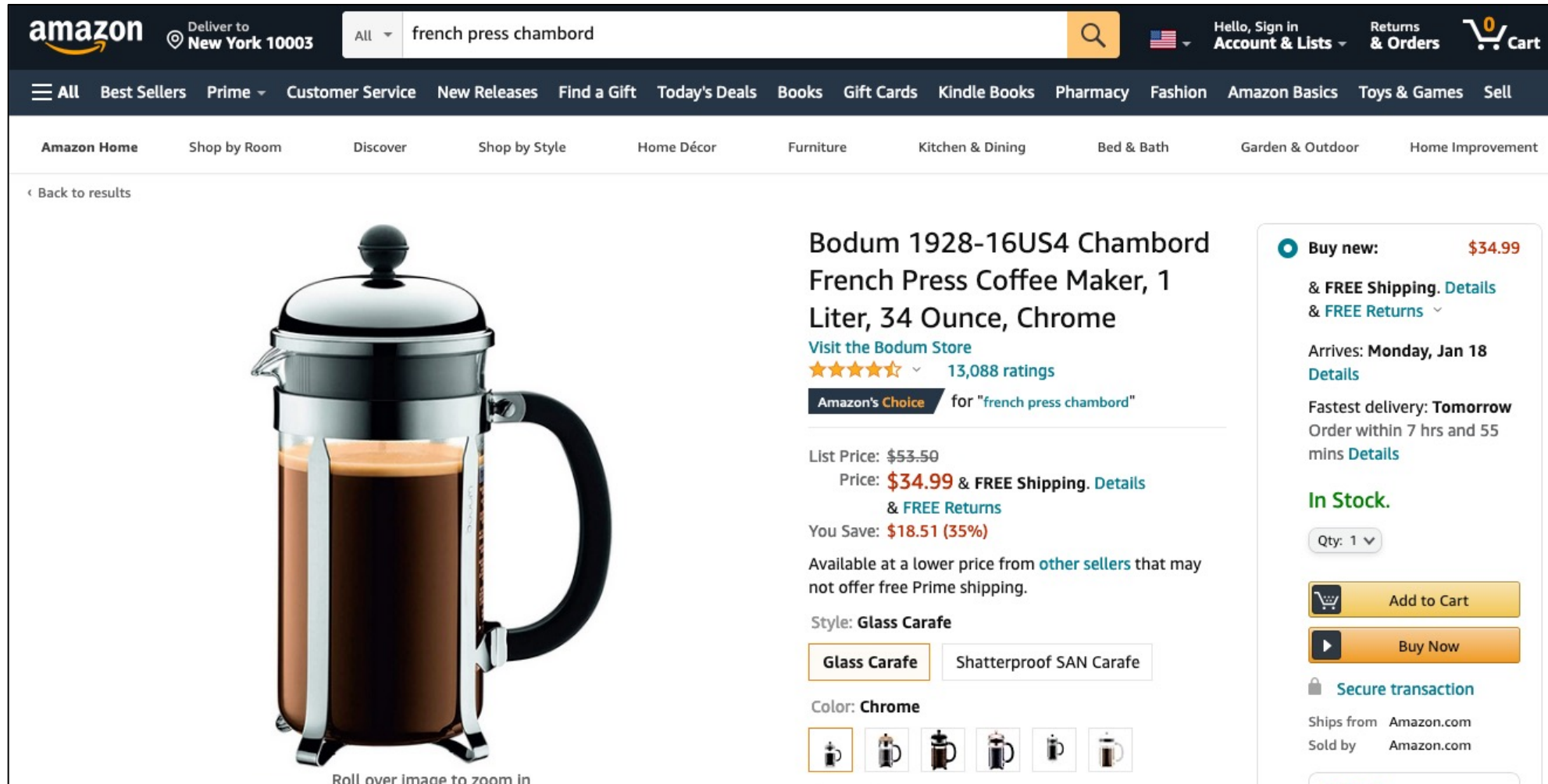
Time-expanded network flow problem



Intractable at Amazon's scale!

Why considering speed?

Why considering speed?




The screenshot shows the Amazon product page for a Bodum 1928-16US4 Chambord French Press Coffee Maker. The page layout includes the Amazon header with the logo, delivery location (New York 10003), and a search bar containing "french press chambord". Below the header is a navigation bar with various categories like "All", "Best Sellers", "Prime", "Customer Service", etc. The main content area features a large image of the coffee maker on the left, which is a chrome-plated glass carafe with a black handle and lid. To the right of the image, the product title "Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome" is displayed, followed by a star rating of 4.5 stars from 13,088 ratings and an "Amazon's Choice" badge. The price section shows a list price of \$53.50, a current price of \$34.99 with free shipping and returns, and a savings of \$18.51 (35%). Below this, it states the product is available at a lower price from other sellers. The "Style" section offers two options: "Glass Carafe" (selected) and "Shatterproof SAN Carafe". The "Color" section shows "Chrome" as the selected option, with a row of six small thumbnail images showing different views and colors of the product. On the right side of the page, a summary box indicates the item is "In Stock", can be bought for \$34.99 with free shipping and returns, and will arrive on Monday, Jan 18. It also shows the fastest delivery is tomorrow if ordered within 7 hours and 55 minutes. At the bottom of this box are buttons for "Add to Cart" and "Buy Now", along with a "Secure transaction" badge and shipping/seller information (Amazon.com).

amazon Deliver to New York 10003 All french press chambord

Amazon Home Shop by Room Discover Shop by Style Home Décor Furniture Kitchen & Dining Bed & Bath Garden & Outdoor Home Improvement

Back to results



Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome

Visit the Bodum Store

★★★★★ 13,088 ratings

Amazon's Choice for "french press chambord"

List Price: \$53.50

Price: **\$34.99** & FREE Shipping. Details & FREE Returns

You Save: **\$18.51 (35%)**

Available at a lower price from [other sellers](#) that may not offer free Prime shipping.

Style: Glass Carafe

Glass Carafe Shatterproof SAN Carafe

Color: Chrome

Roll over image to zoom in

Buy new: **\$34.99**

& FREE Shipping. Details & FREE Returns

Arrives: **Monday, Jan 18** Details

Fastest delivery: **Tomorrow** Order within 7 hrs and 55 mins Details

In Stock.

Qty: 1

Add to Cart

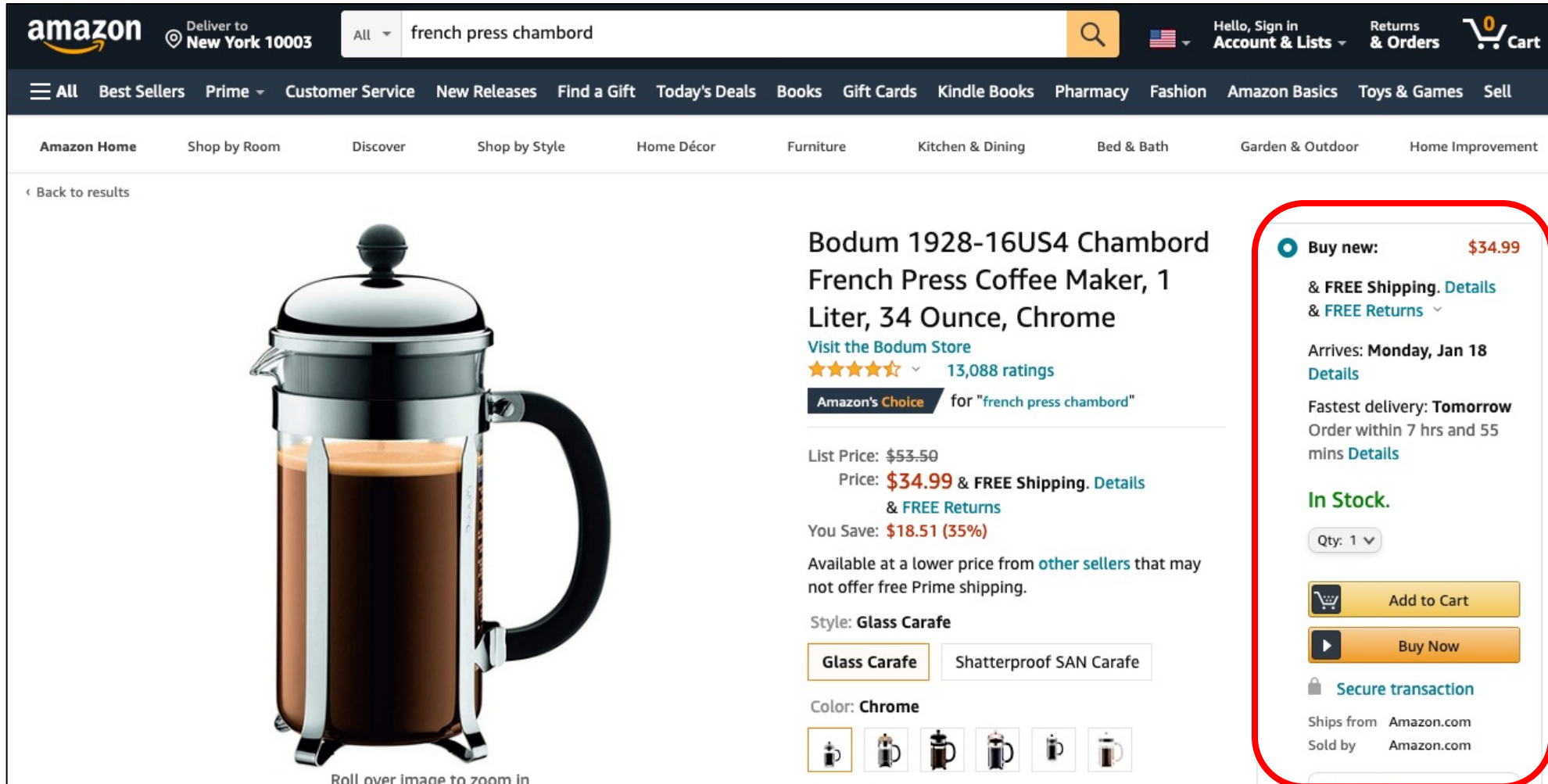
Buy Now

Secure transaction

Ships from Amazon.com

Sold by Amazon.com

Why considering speed?




The image shows a screenshot of an Amazon product page for a Bodum French press coffee maker. The page layout includes a top navigation bar with the Amazon logo, delivery location (New York 10003), a search bar containing 'french press chambord', and links for account, orders, and cart. Below this is a category navigation bar with options like 'All', 'Best Sellers', 'Prime', etc. The main content area features a large image of the coffee maker on the left, with a zoom-in prompt at the bottom. To the right of the image is the product title, 'Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome', followed by a star rating of 4.5 and 13,088 ratings. Below the rating is an 'Amazon's Choice' badge. The price section shows a list price of \$53.50, a current price of \$34.99, and a 35% discount. A note mentions that other sellers may offer a lower price. The product is described as 'Glass Carafe' and 'Chrome'. On the right side, a red-bordered box highlights the purchase options: 'Buy new: \$34.99' with free shipping and returns, an arrival date of Monday, Jan 18, and a 'Fastest delivery: Tomorrow' option. It also shows the quantity set to 1, 'Add to Cart' and 'Buy Now' buttons, a 'Secure transaction' badge, and shipping/selling information from Amazon.com.

amazon Deliver to New York 10003 All french press chambord Hello, Sign in Account & Lists Returns & Orders Cart

All Best Sellers Prime Customer Service New Releases Find a Gift Today's Deals Books Gift Cards Kindle Books Pharmacy Fashion Amazon Basics Toys & Games Sell

Amazon Home Shop by Room Discover Shop by Style Home Décor Furniture Kitchen & Dining Bed & Bath Garden & Outdoor Home Improvement

Back to results



Roll over image to zoom in

Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome

Visit the Bodum Store

★★★★★ 13,088 ratings

Amazon's Choice for "french press chambord"

List Price: \$53.50

Price: **\$34.99** & FREE Shipping. Details & FREE Returns

You Save: **\$18.51 (35%)**

Available at a lower price from [other sellers](#) that may not offer free Prime shipping.

Style: **Glass Carafe**

Glass Carafe Shatterproof SAN Carafe

Color: **Chrome**

Buy new: **\$34.99**

& FREE Shipping. Details & FREE Returns

Arrives: **Monday, Jan 18** Details

Fastest delivery: **Tomorrow** Order within 7 hrs and 55 mins Details

In Stock.

Qty: 1

Add to Cart

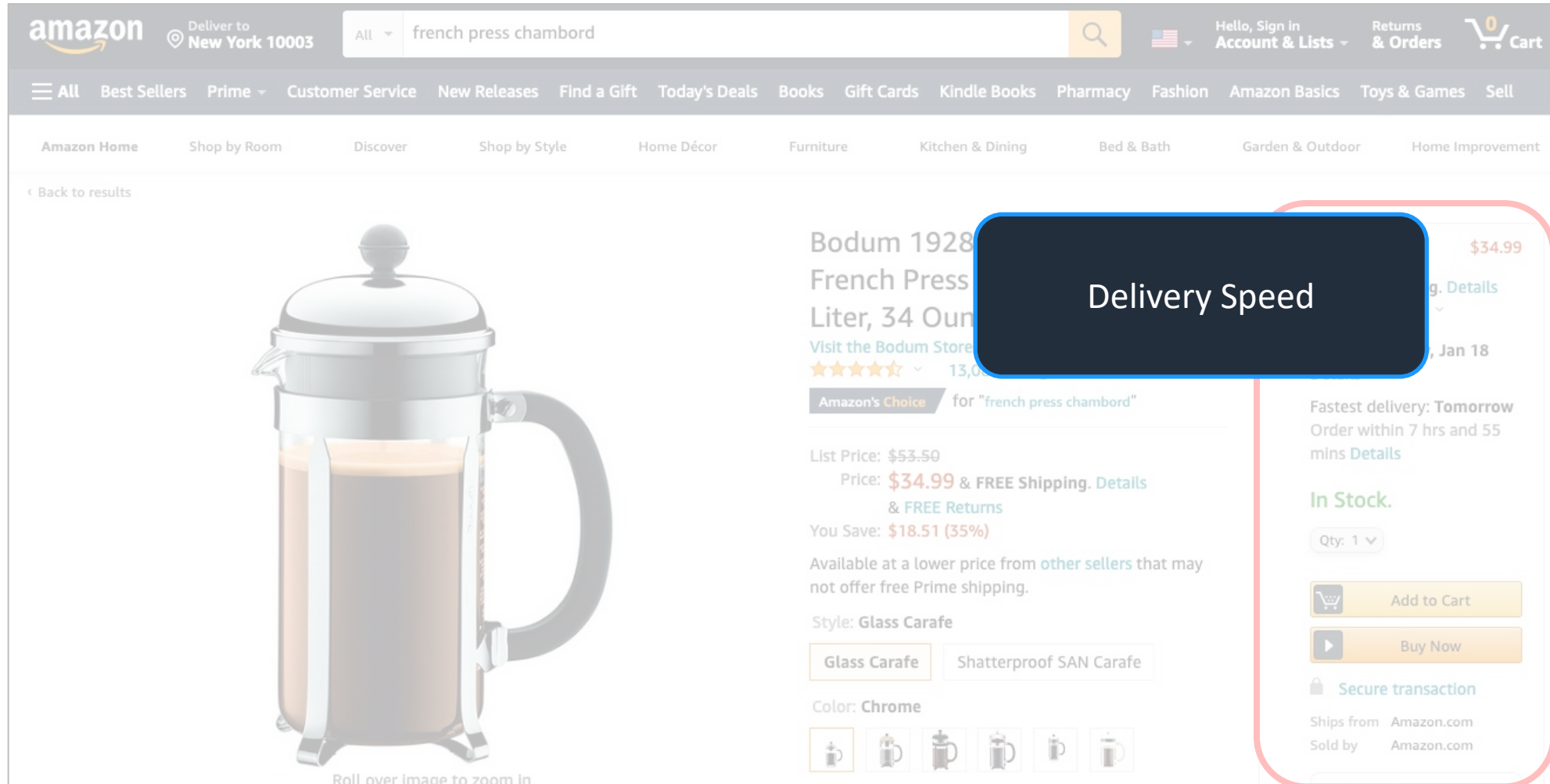
Buy Now

Secure transaction

Ships from Amazon.com

Sold by Amazon.com

Why considering speed?



amazon Deliver to New York 10003 All french press chambord

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Amazon Home Shop by Room Discover Shop by Style Home Décor Furniture Kitchen & Dining Bed & Bath Garden & Outdoor Home Improvement

Back to results

Bodum 1928 French Press, 34 Ounces, 1 Liter, 34 Ounces

Visit the Bodum Store

★★★★★ 13,000+ reviews

Amazon's Choice for "french press chambord"

List Price: \$53.50
Price: **\$34.99** & FREE Shipping. Details
& FREE Returns
You Save: **\$18.51 (35%)**

Available at a lower price from other sellers that may not offer free Prime shipping.

Style: Glass Carafe

Glass Carafe Shatterproof SAN Carafe

Color: Chrome

Roll over image to zoom in

Delivery Speed

Fastest delivery: **Tomorrow**
Order within 7 hrs and 55 mins Details

In Stock.

Qty: 1

Add to Cart

Buy Now

Secure transaction

Ships from Amazon.com
Sold by Amazon.com

Why considering speed?

The screenshot shows the Amazon product page for a Bodum 1928 French Press. The page layout includes the Amazon logo, delivery location (New York 10003), search bar, and navigation links. The product title is "Bodum 1928 French Press, 34 Ounces, 1 Liter, 34 Ounces". The price is \$34.99, and it is marked as "Amazon's Choice". The delivery information states "Fastest delivery: Tomorrow" and "Order within 7 hrs and 55 mins". The product is "In Stock".

Two callout boxes are overlaid on the image:

- Delivery Speed**: A dark blue box with white text, pointing to the delivery information section.
- Orders and customer satisfaction**: A dark blue box with white text, pointing to the product image and the "Amazon's Choice" badge.

The product image shows a silver French press with a black handle and a black lid. The text "Roll over image to zoom in" is visible at the bottom of the image.

Why considering speed?

The image is a screenshot of an Amazon product page for a Bodum 1928 French Press. The page features a search bar at the top with the text "french press chambord". Below the search bar, there are navigation links for various categories like "All", "Best Sellers", "Prime", etc. The product itself is a silver and glass French press, shown in a large image on the left. To the right of the image, the product title "Bodum 1928 French Press" is visible, along with its price "\$34.99" and a "Fastest delivery: Tomorrow" badge. The page also includes a "Back to results" link, a "Roll over image to zoom in" prompt, and a "Secure transaction" badge. Three dark blue callout boxes with orange arrows are overlaid on the page: "Network design" points to the product image, "Delivery Speed" points to the delivery badge, and "Orders and customer satisfaction" points to the product title and price area.

amazon Deliver to New York 10003 All french press chambord

Best Sellers Prime Customer Service New Releases Find a Gift Today's Deals Books Gift Cards Kindle Books Pharmacy Fashion Amazon Basics Toys & Games Sell

Amazon Home Shop by Room Discover Shop by Style Home Décor Furniture Kitchen & Dining Bed & Bath Garden & Outdoor Home Improvement

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Network design

Bodum 1928 French Press
Liter, 34 Ounces
Visit the Bodum Store
★★★★★ 13,000+ reviews

Amazon's Choice for "french press chambord"

List Price: \$53.59
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Delivery Speed

Fastest delivery: **Tomorrow**
Order within 7 hrs and 55 mins [Details](#)

In Stock.
Qty: 1

Add to Cart Buy Now

Secure transaction

Ships from Amazon.com
Sold by Amazon.com

Orders and customer satisfaction

Roll over image to zoom in

Why considering speed?

The image shows a screenshot of an Amazon product page for a Bodum 1928 French Press. The page includes the Amazon logo, delivery location (New York 10003), search bar, and navigation links. The product is a silver and white French press, currently priced at \$34.99 (a 35% discount from the list price of \$53.59). It is marked as 'Amazon's Choice' and 'In Stock'. The fastest delivery is 'Tomorrow' if ordered within 7 hours and 55 minutes. The page also shows 'Add to Cart' and 'Buy Now' buttons, and a 'Secure transaction' badge.

Overlaid on the page are three dark blue boxes with white text, connected by orange arrows:

- Network design**: A box on the left with an arrow pointing to the product image.
- Delivery Speed**: A box on the right with an arrow pointing to the 'Fastest delivery' information.
- Orders and customer satisfaction**: A box at the bottom with an arrow pointing to the product image.

The arrows indicate a flow from 'Network design' to 'Delivery Speed' and from 'Orders and customer satisfaction' to 'Delivery Speed', suggesting that these factors are interconnected in the context of e-commerce performance.

Why considering speed?



Why considering speed?

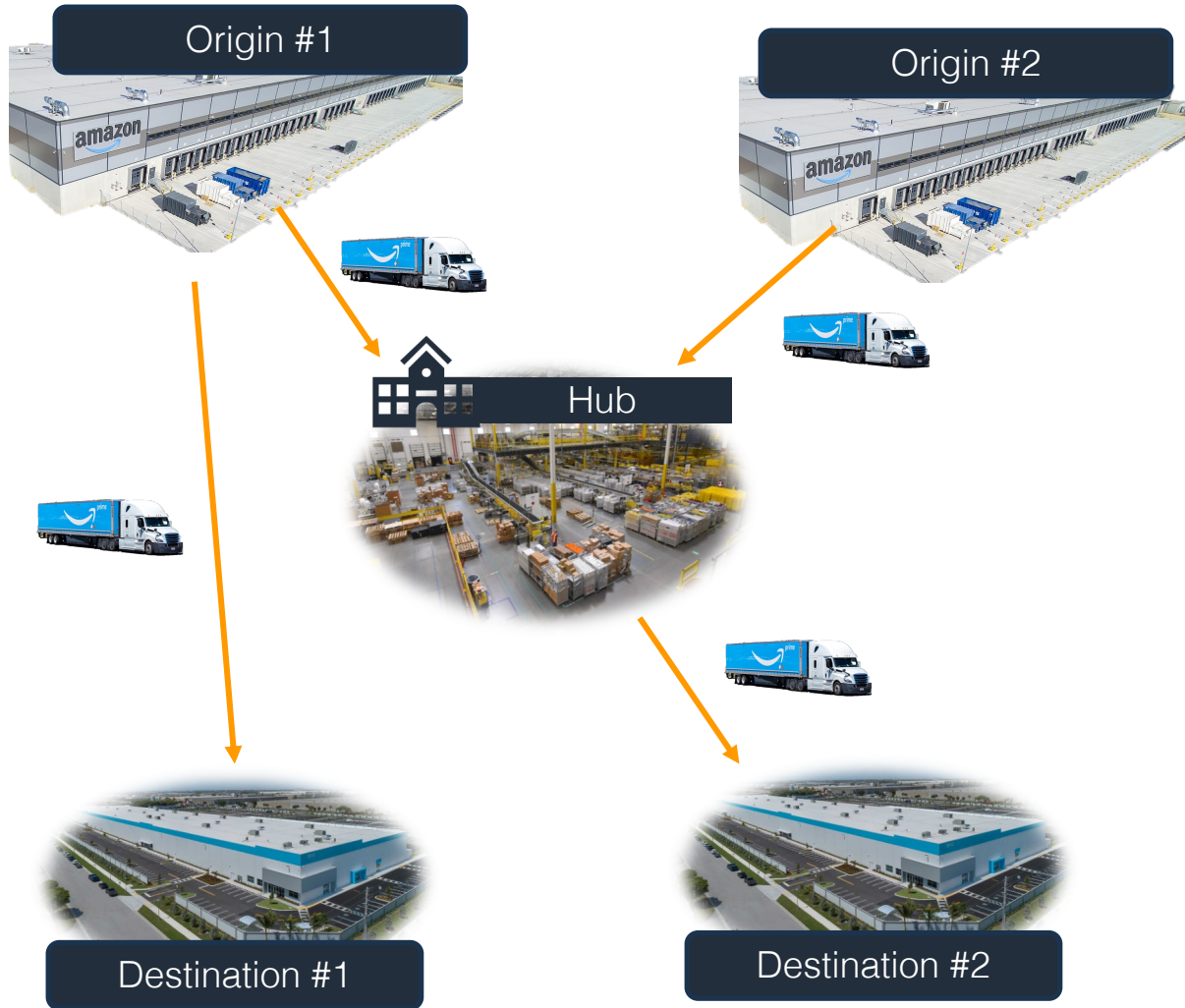
Key objective: Design the cheapest and fastest network for our customers



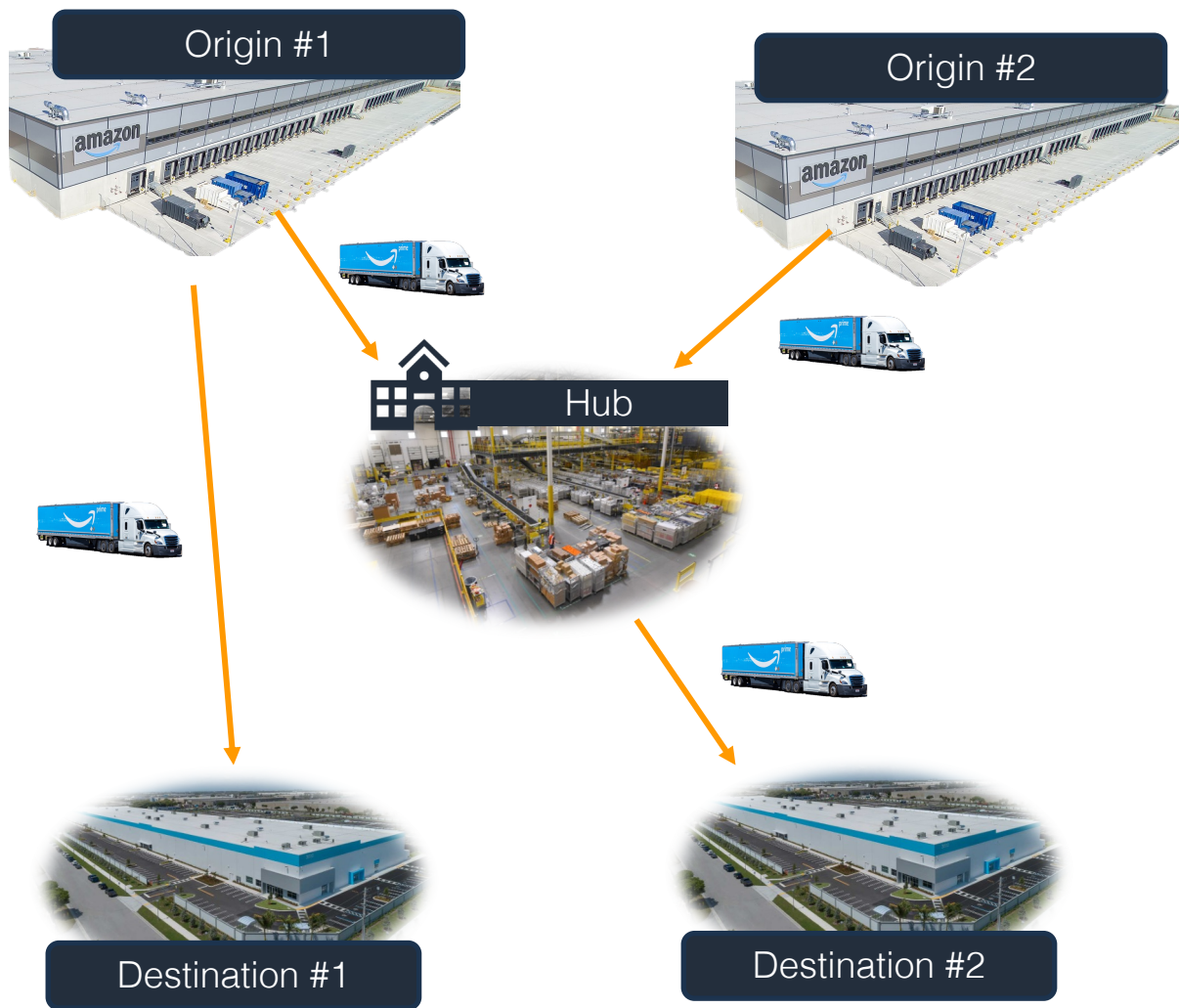
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Network Design: Connectivity

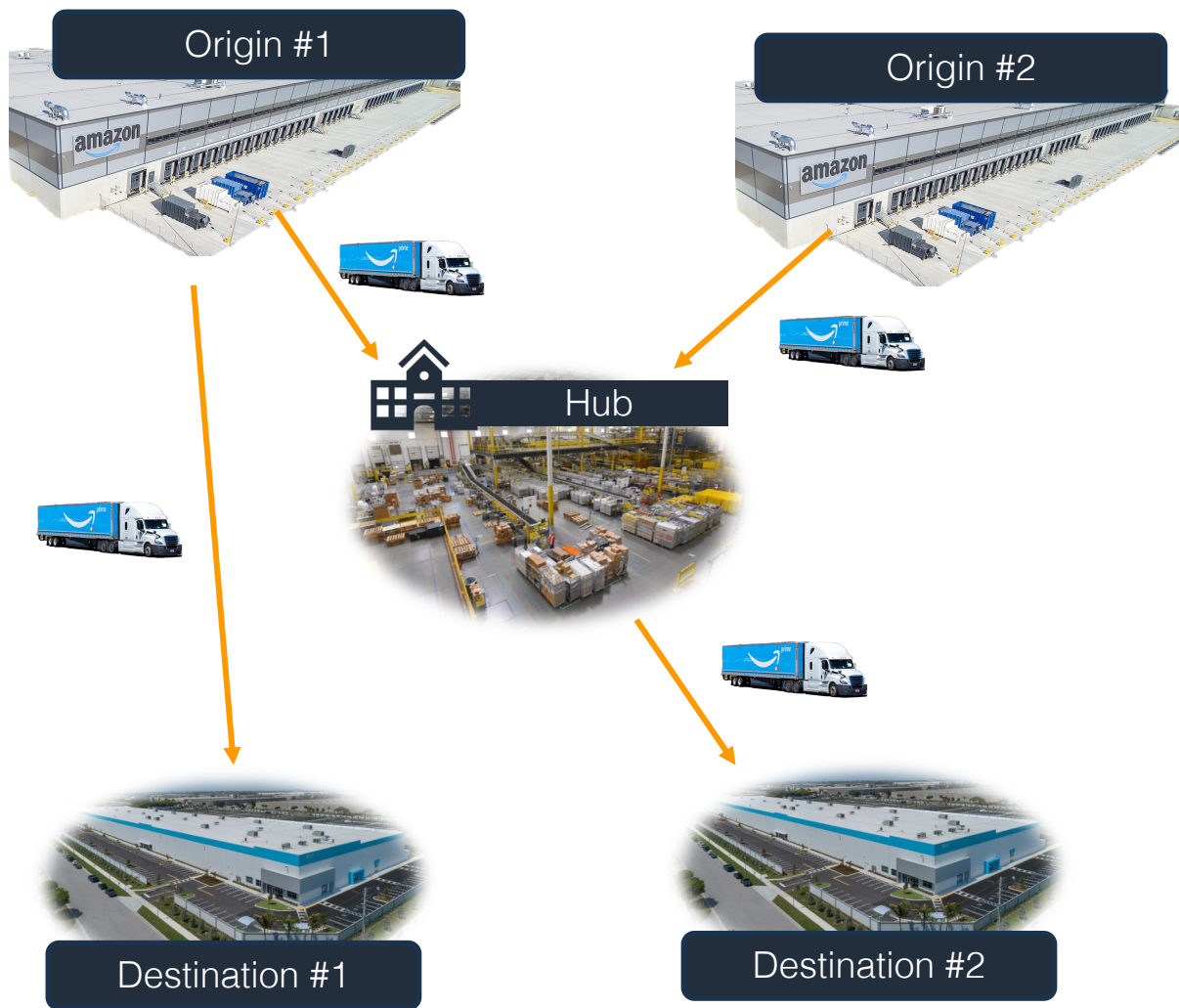


Network Design: Connectivity



$$\begin{aligned} \min_{\mathbf{p}, \mathbf{v}} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) \\ \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \end{aligned}$$

Network Design: Connectivity

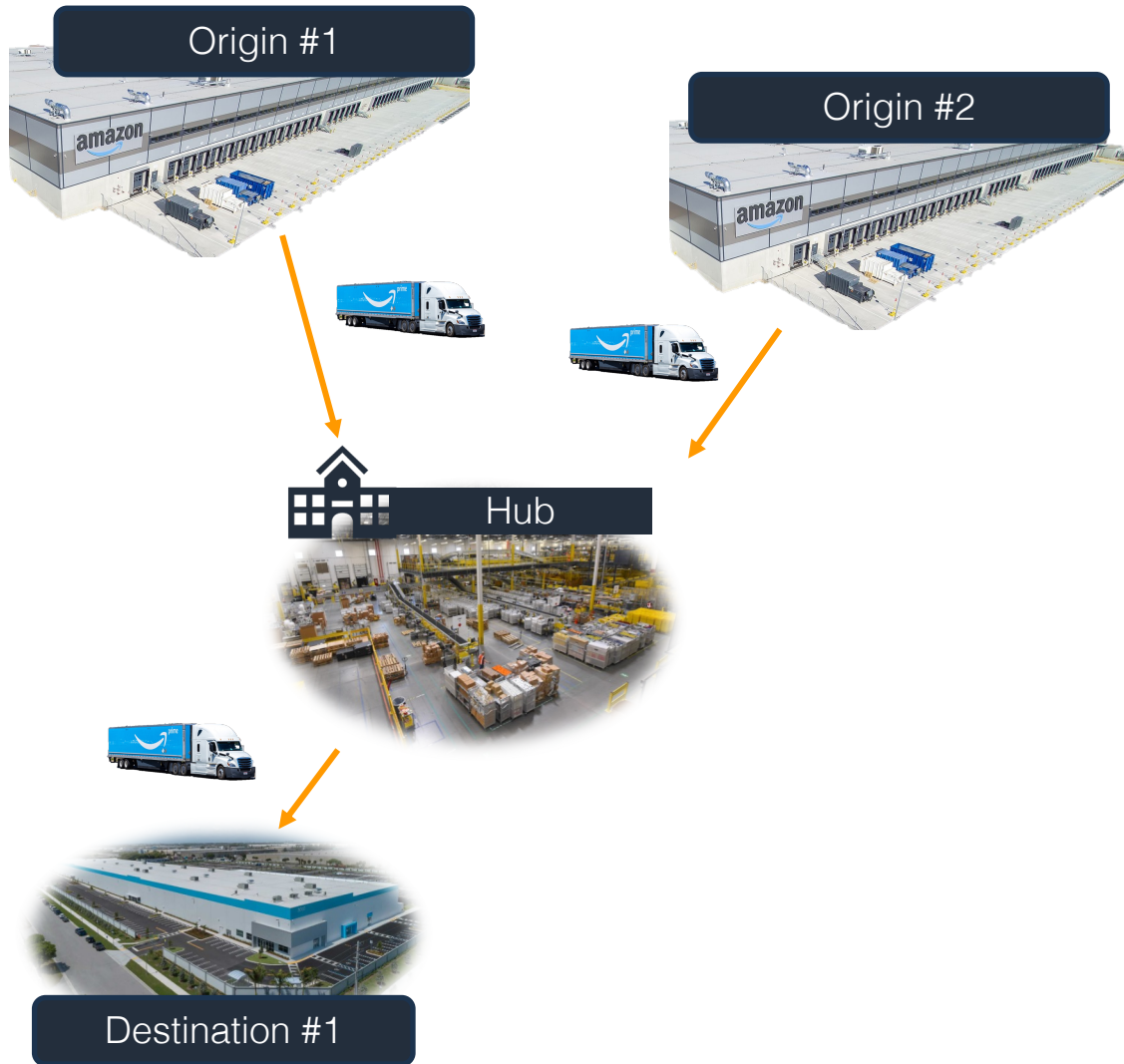


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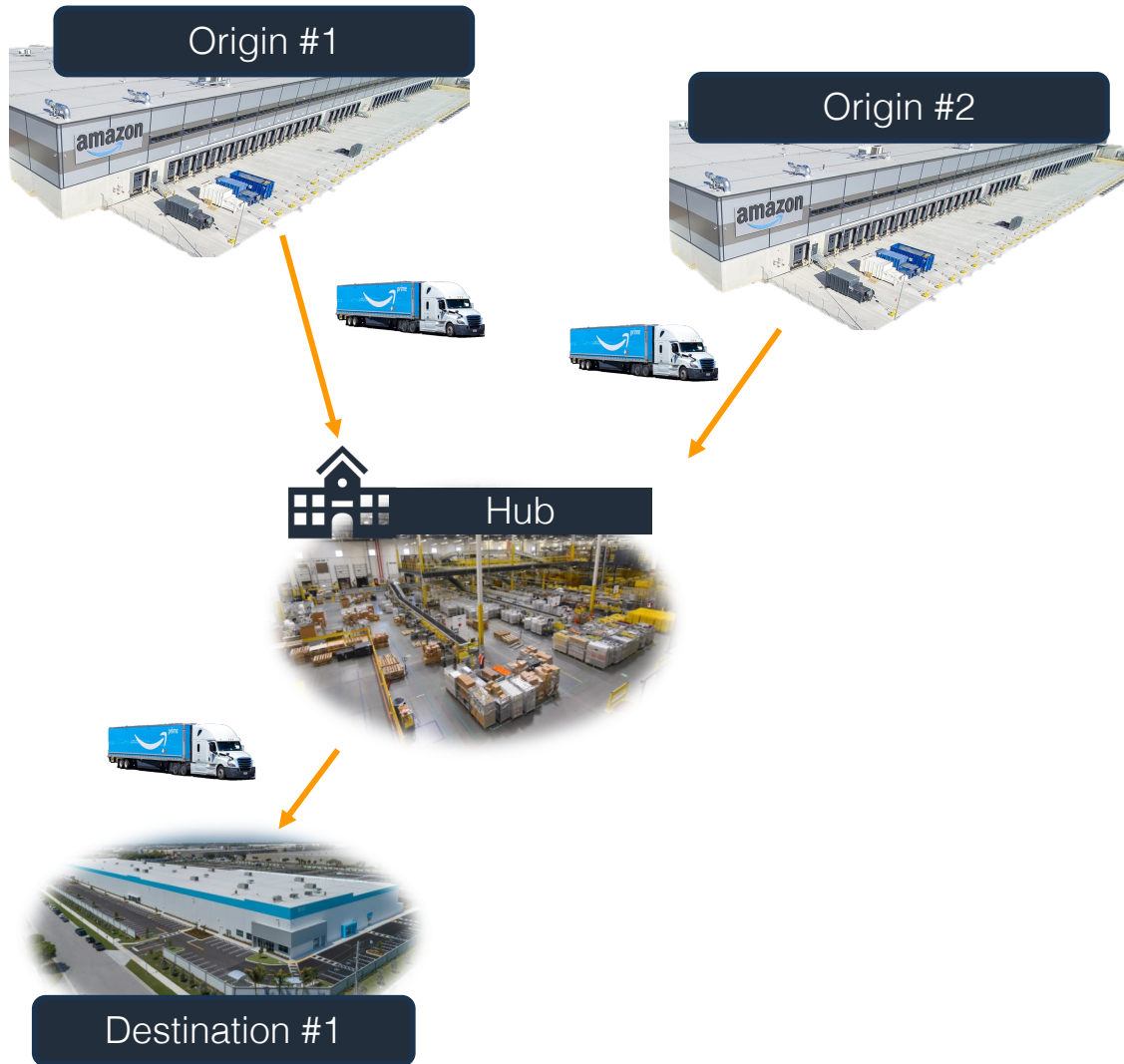
Path vector variable

Trucks vector variable

Network Design: Timing



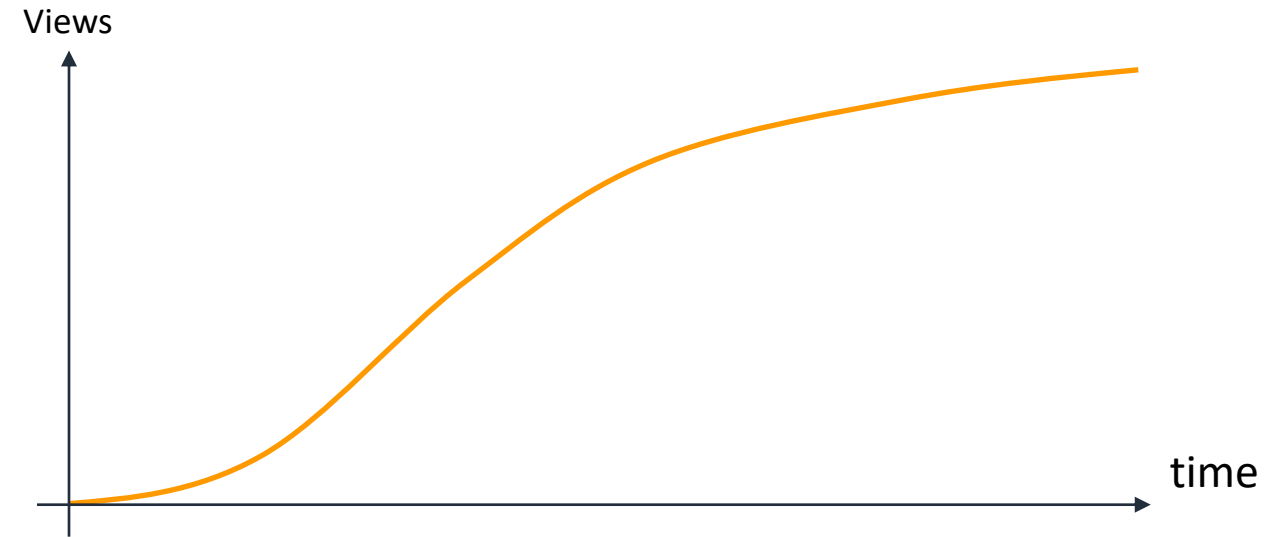
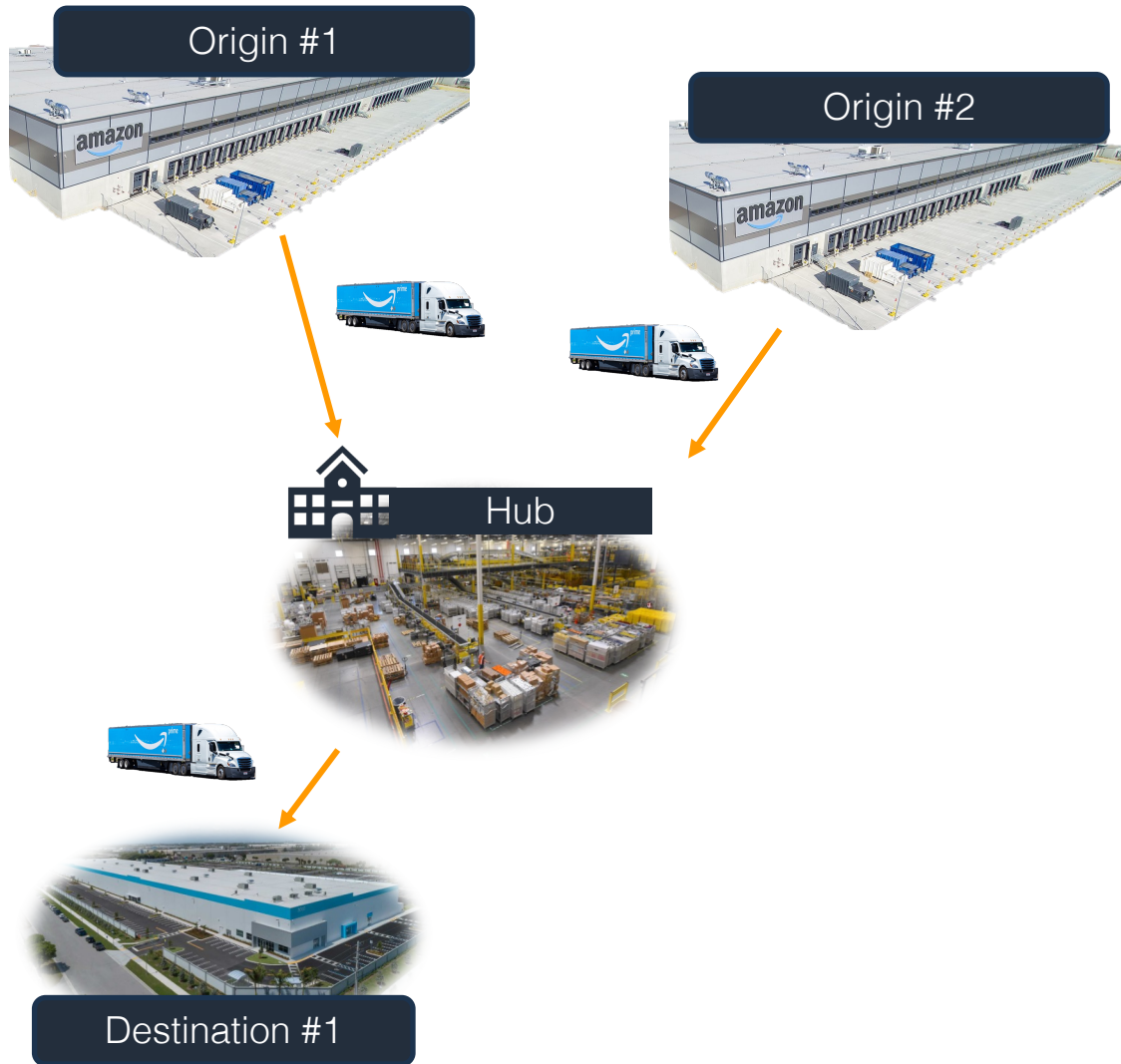
Network Design: Timing



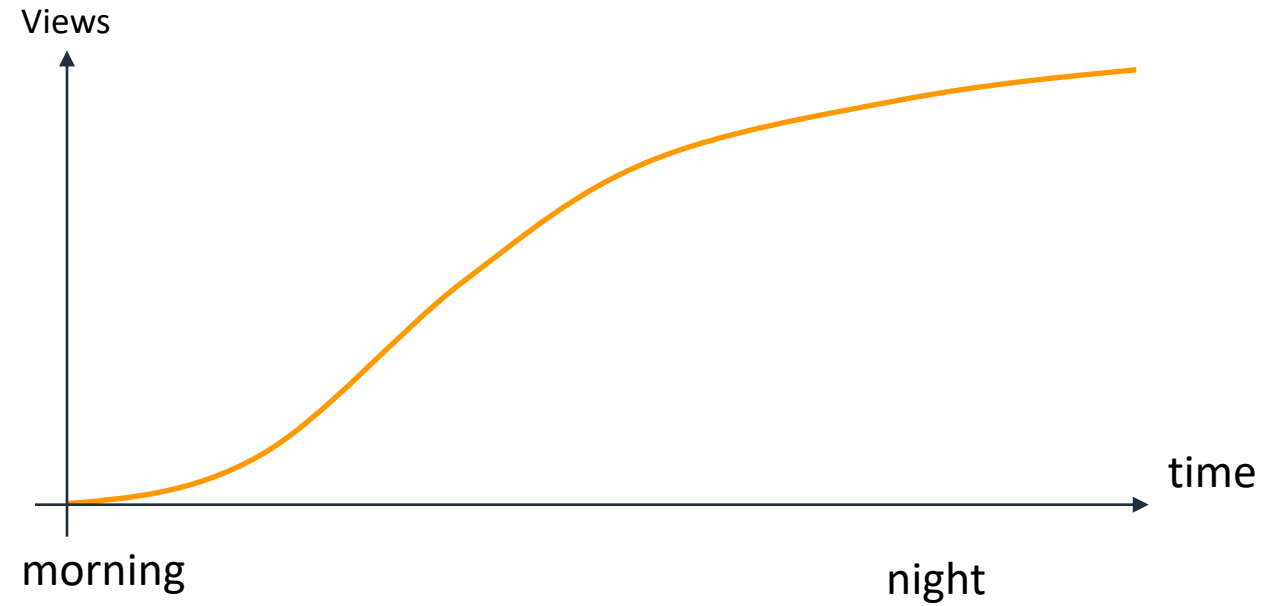
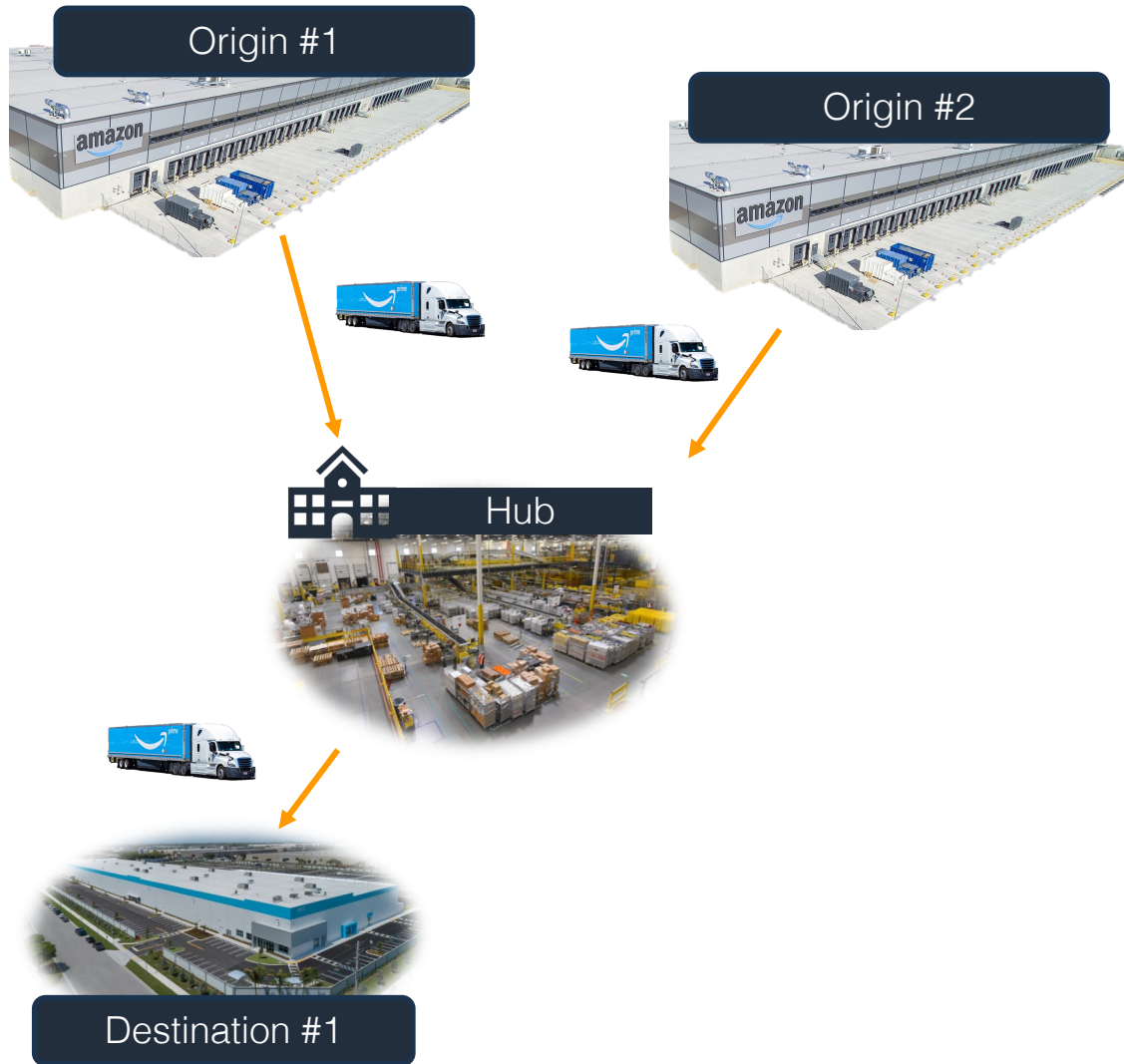
Views

time

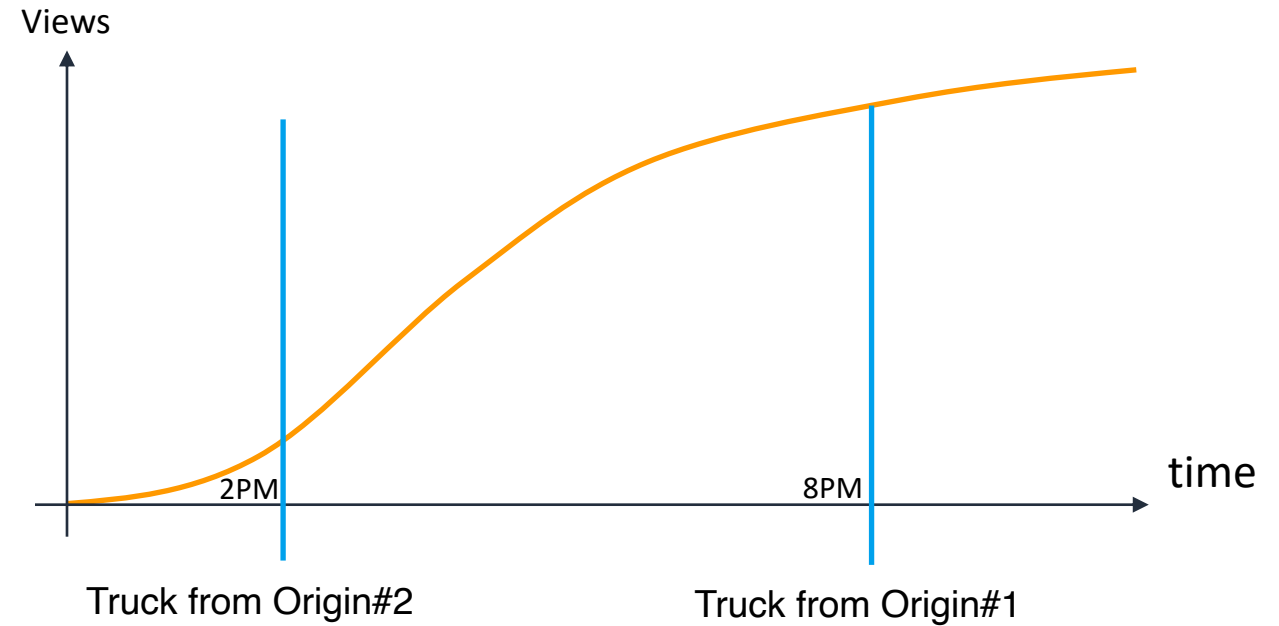
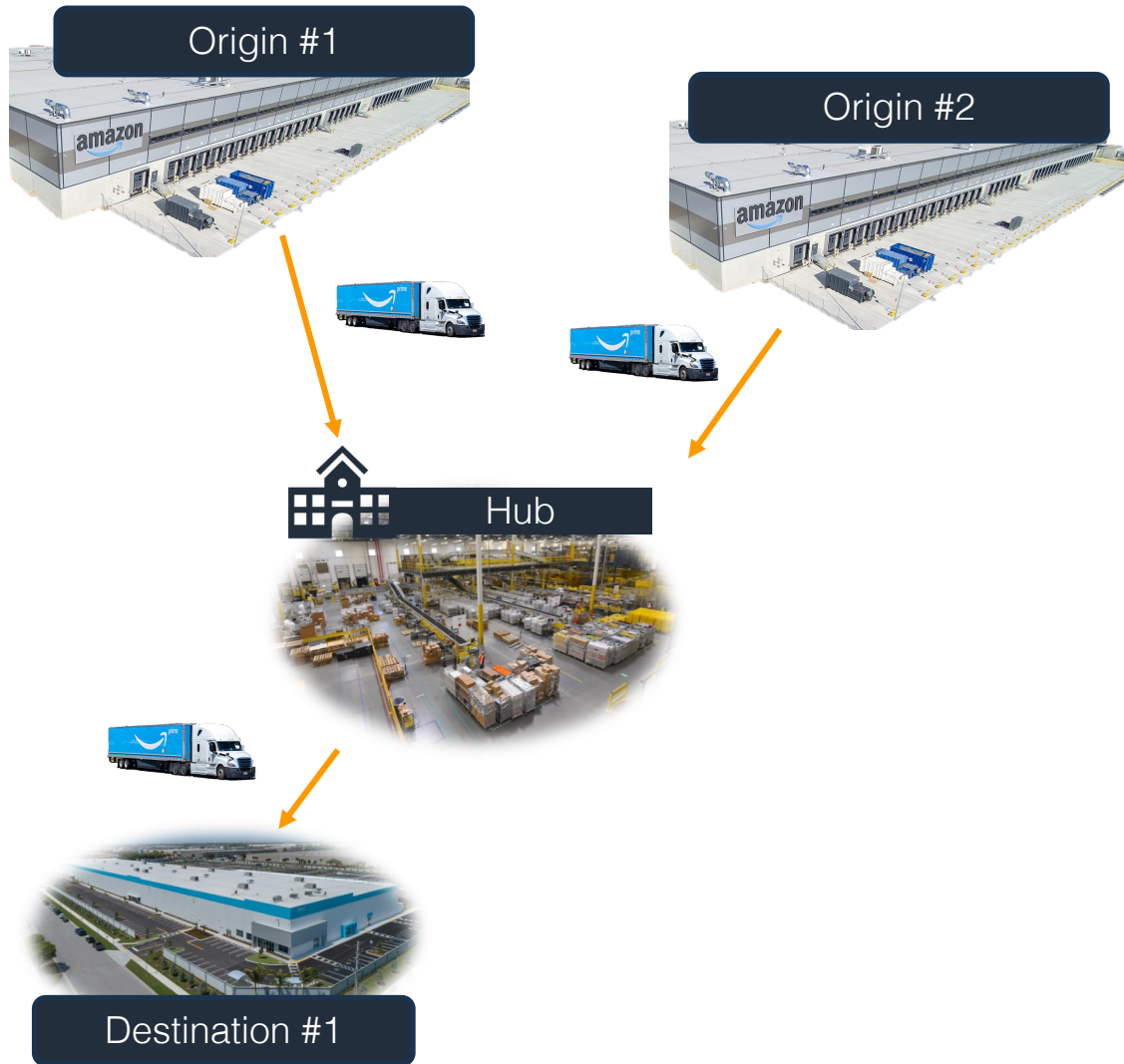
Network Design: Timing



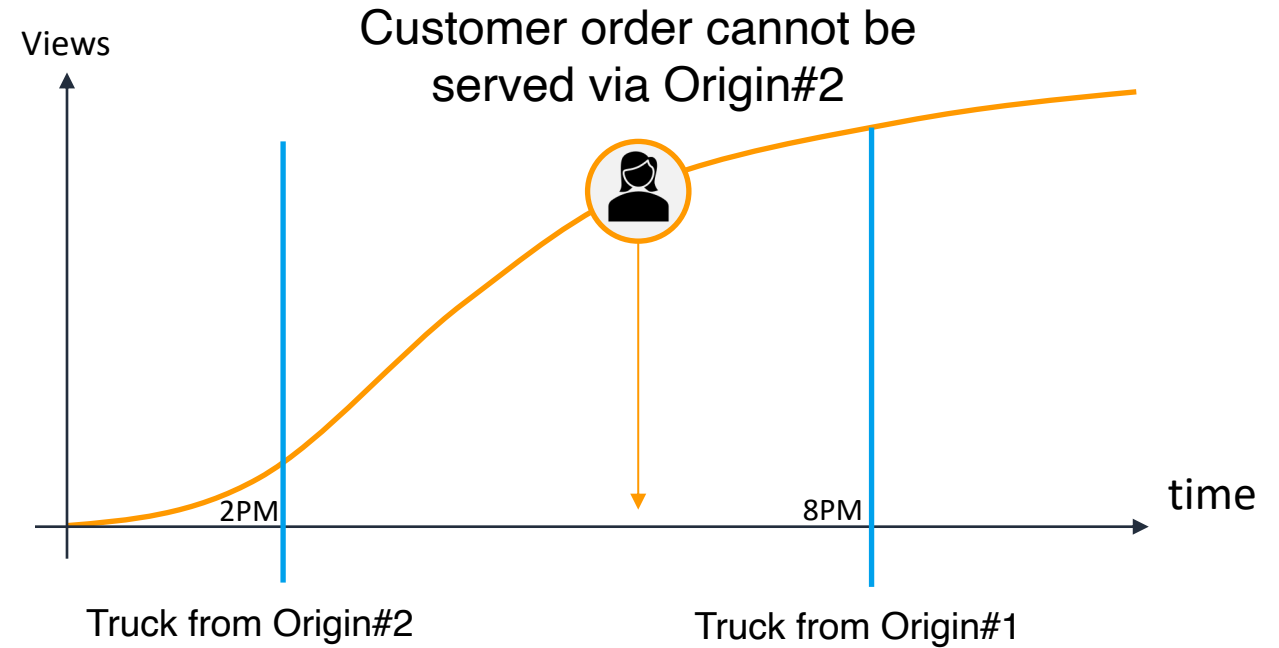
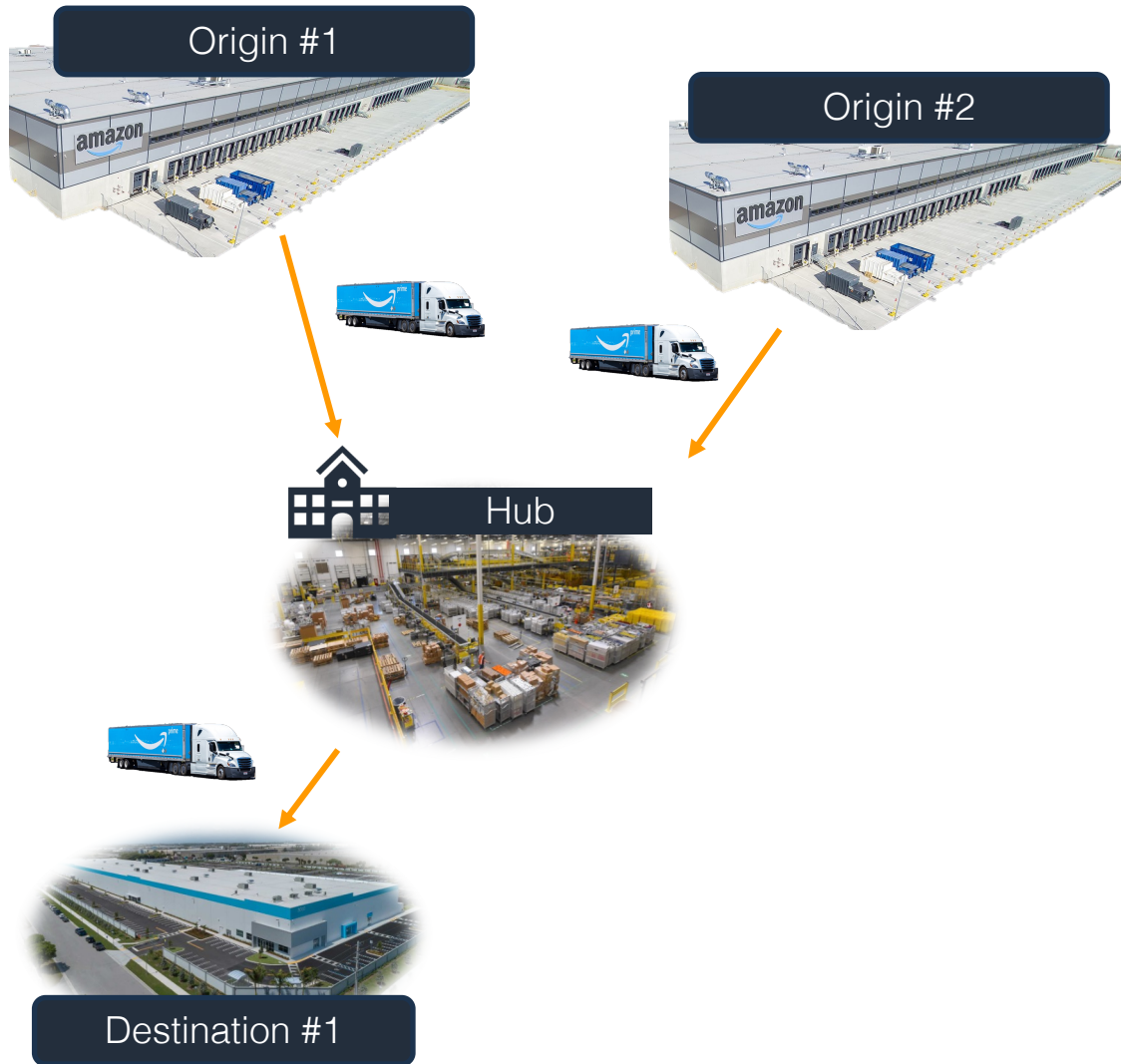
Network Design: Timing



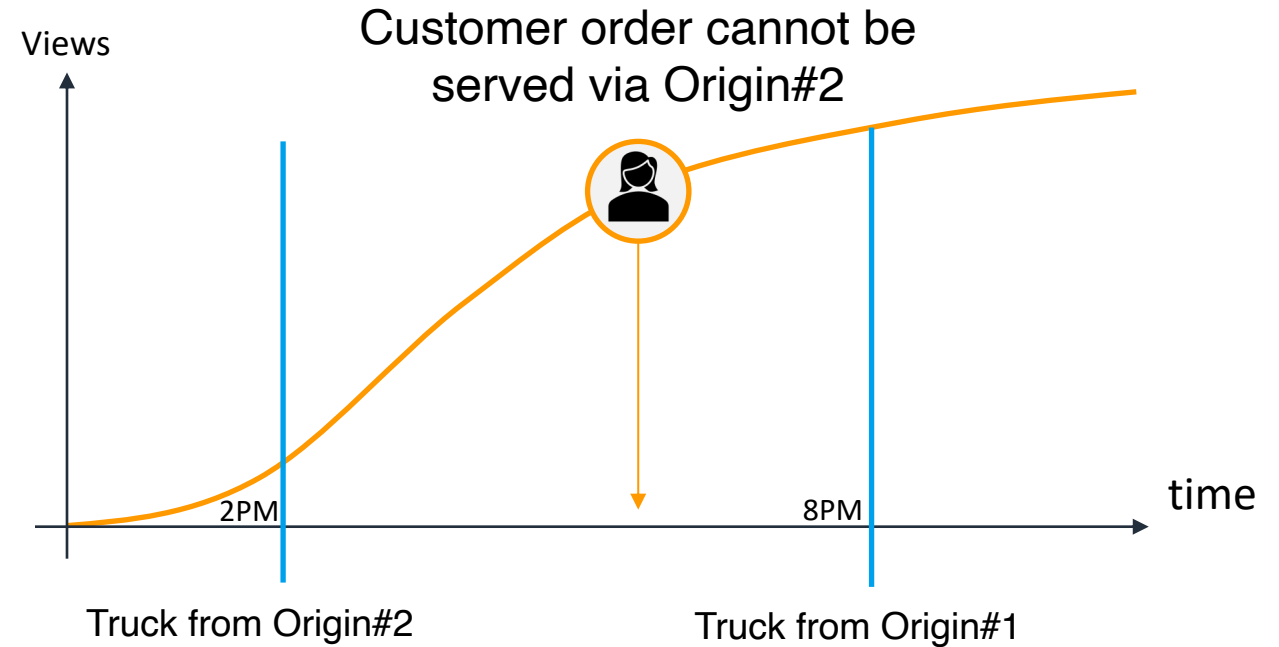
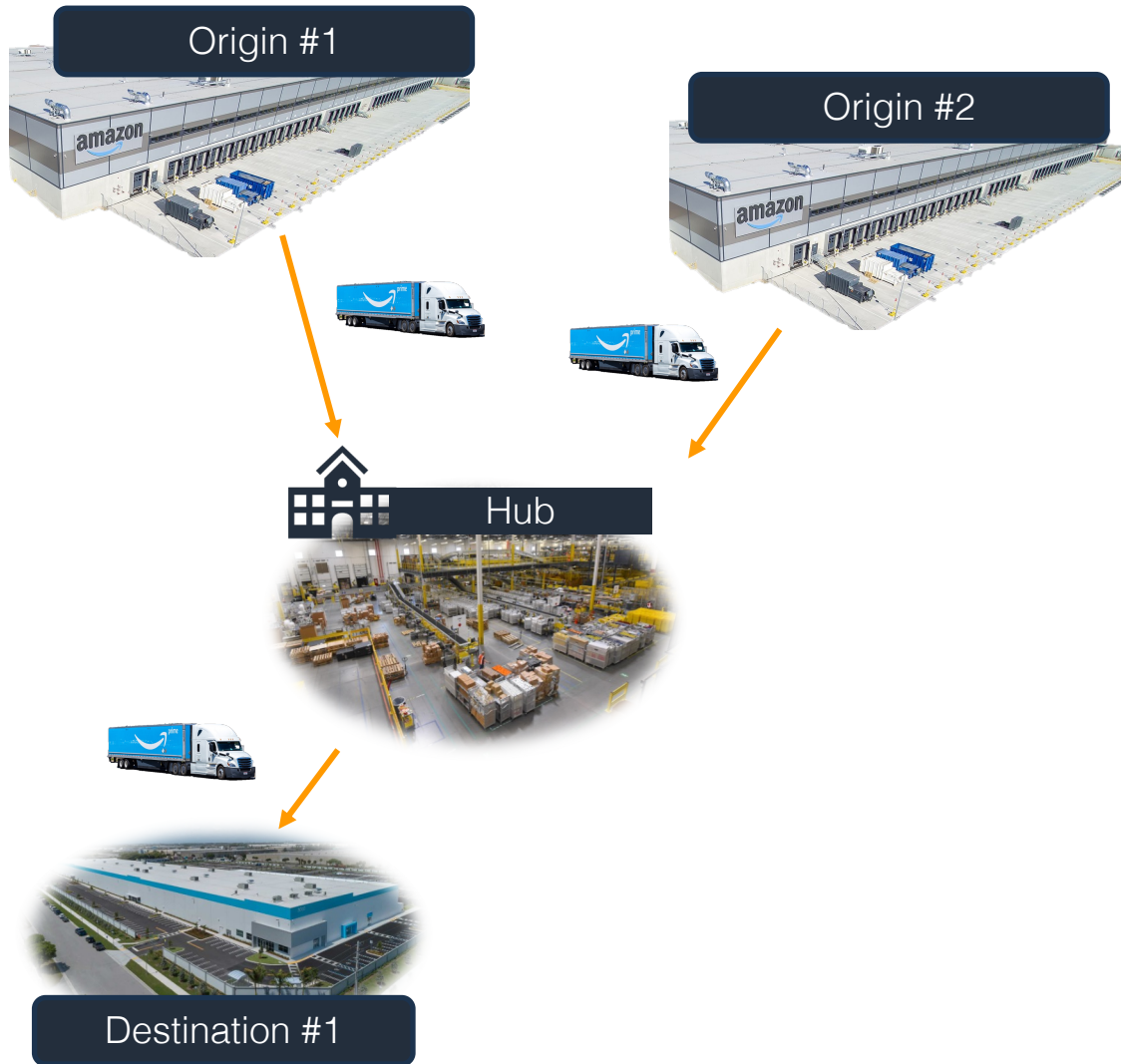
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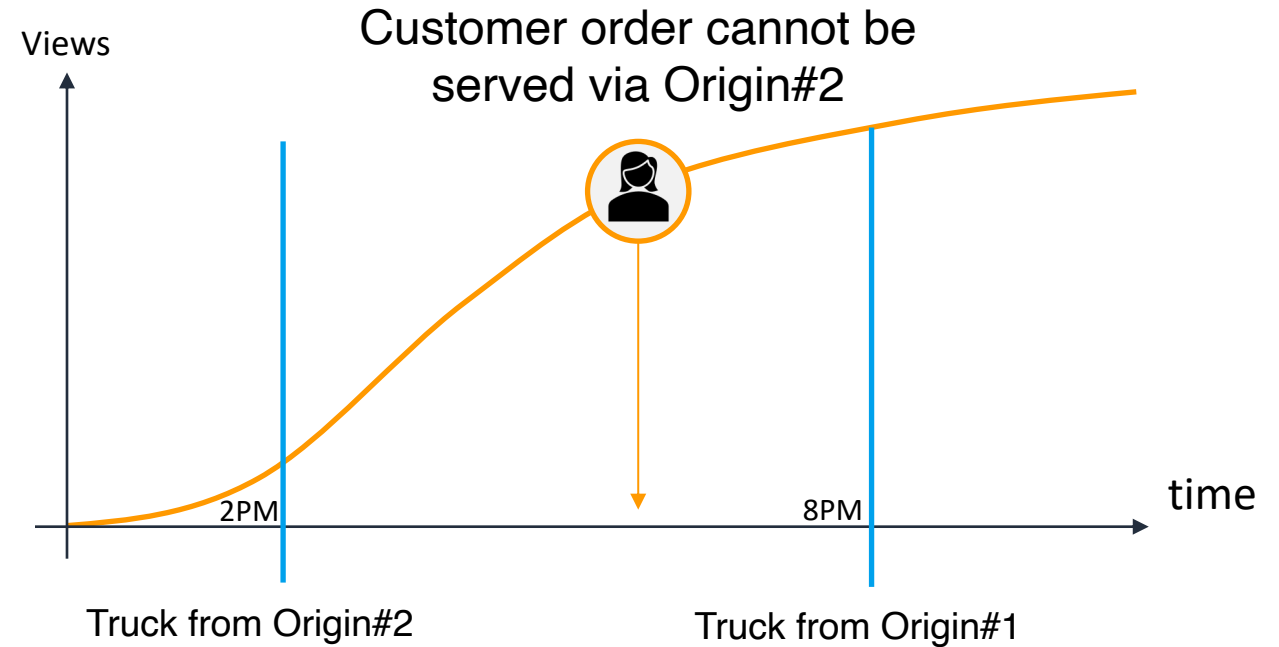
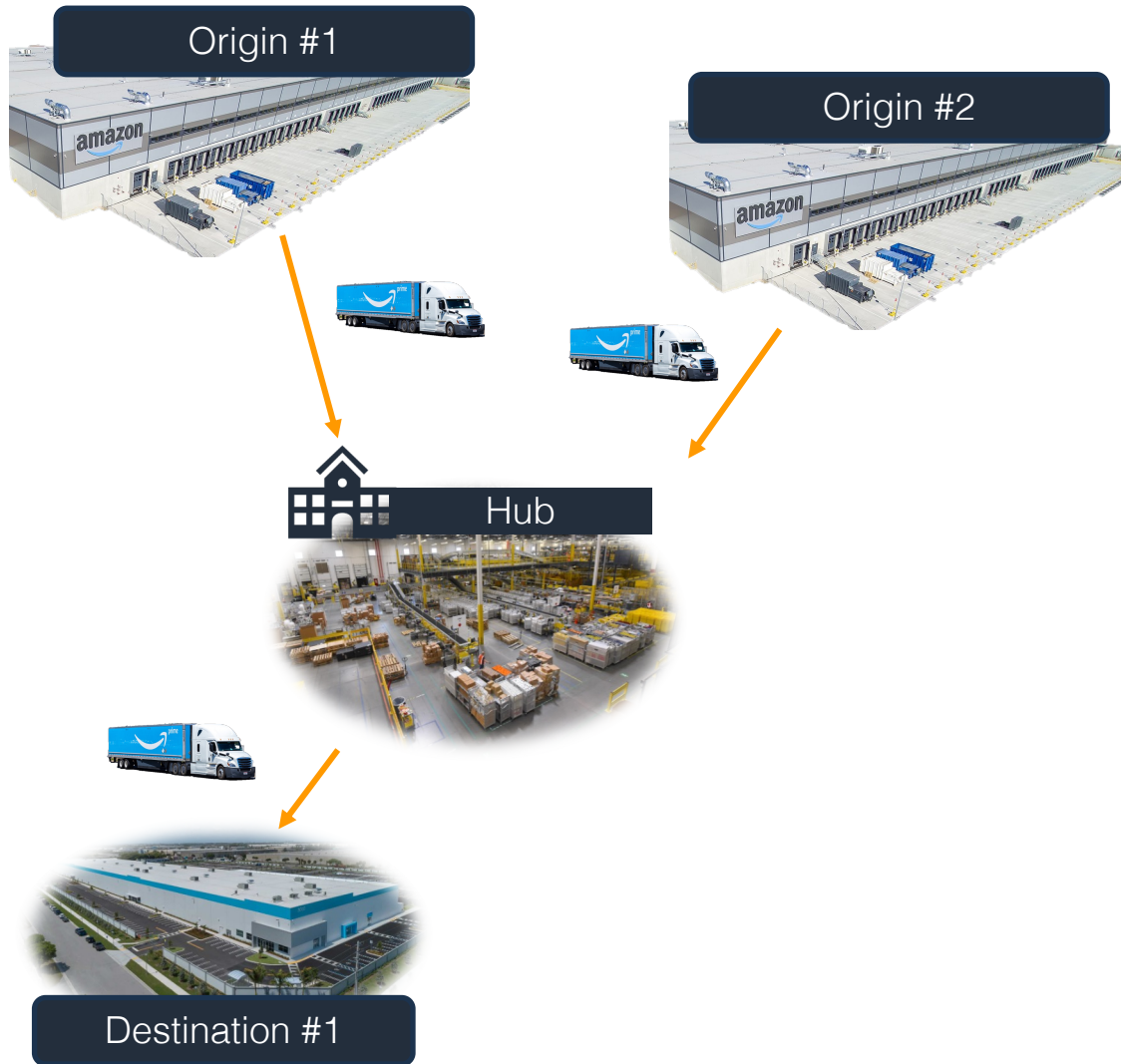


Network Design: Timing



$$\begin{aligned} \max_z \quad & \text{Speed}(z) \\ \text{s.t.} \quad & z \in \text{FeasibleSchedule}(p) \end{aligned}$$

Network Design: Timing

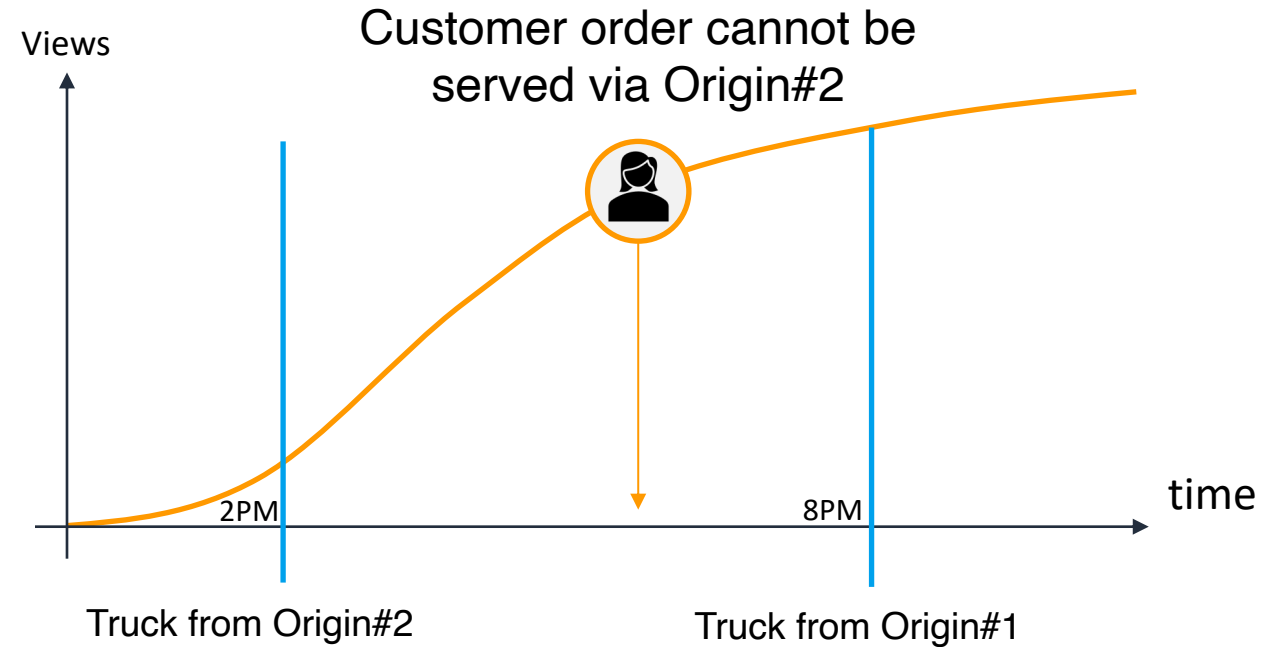
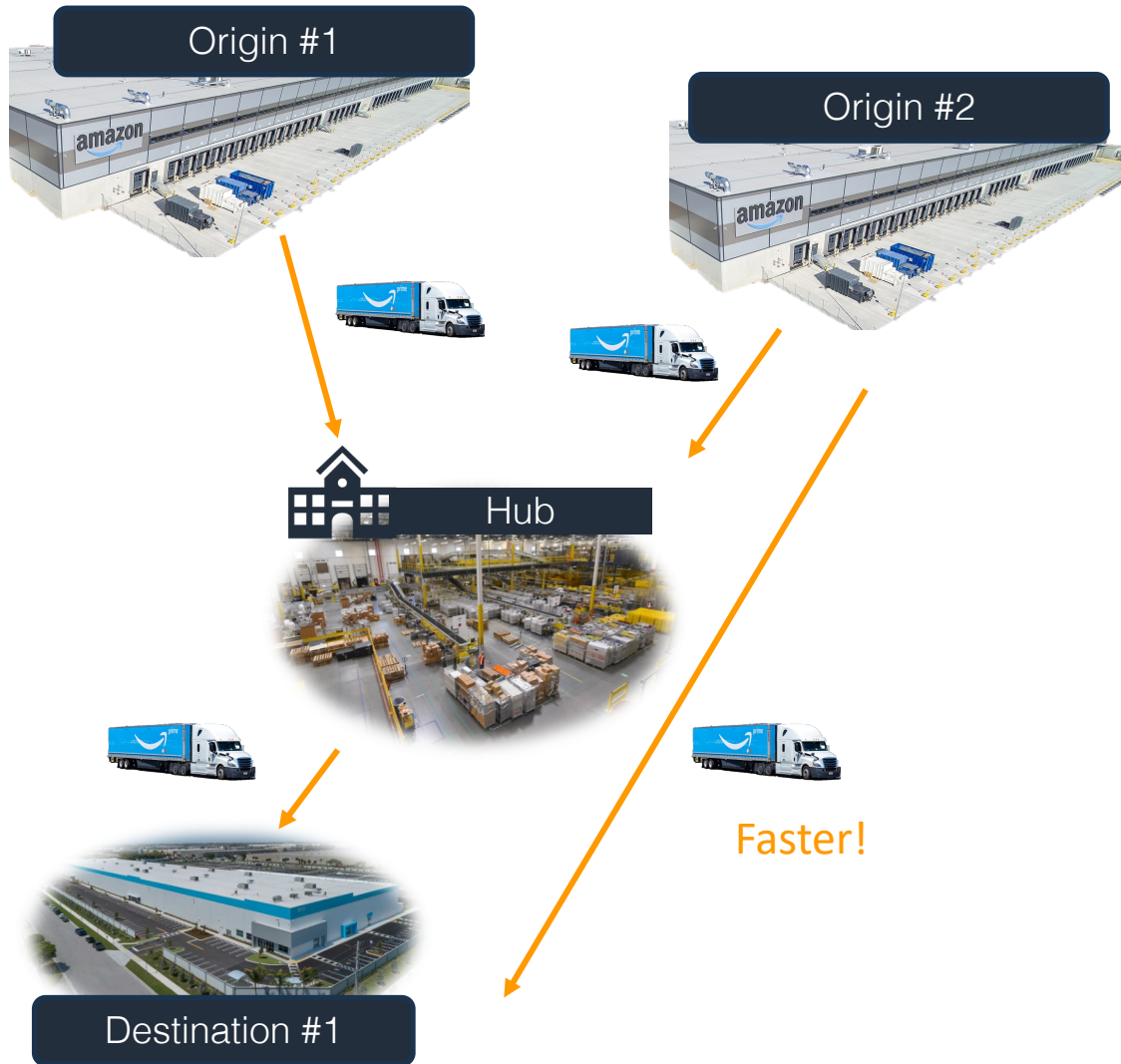


$$\begin{aligned} \max_z \quad & \text{Speed}(z) \\ \text{s.t.} \quad & z \in \text{FeasibleSchedule}(p) \end{aligned}$$

Truck schedule vector variable

Dependency on connectivity

Network Design: Timing



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Truck schedule vector variable

Dependency on connectivity

Network Design: The joint problem

$$\begin{array}{ll} \min_{\mathbf{p}, \mathbf{y}, \mathbf{z}} & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \text{Speed}(\mathbf{z}) \\ \text{s.t.} & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \\ & \mathbf{z} \in \text{FeasibleSchedule}(\mathbf{p}) \end{array}$$

Network Design: The joint problem

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Why is this problem hard to solve?

Network Design: The joint problem

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(a) Feasibility: We must consider granular non-convex operational constraints, e.g., site opening hours.

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- (a) Feasibility: We must consider granular non-convex operational constraints, e.g., site opening hours.
- (b) Speed objective: Inventory at origins impacts the speed given by expensive fast connections.

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(a) Feasibility: We must consider granular non-convex operational constraints, e.g., site opening hours.

(b) Speed objective: Inventory at origins impacts the speed given by expensive fast connections.

(c) Scale: Billions of variables to model hourly decisions, e.g., when a truck should depart.

Network Design: The joint problem

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- (d) Uncertainty: Customers' demand is uncertain, thus we should minimize the expected cost.

Network Design: The joint problem

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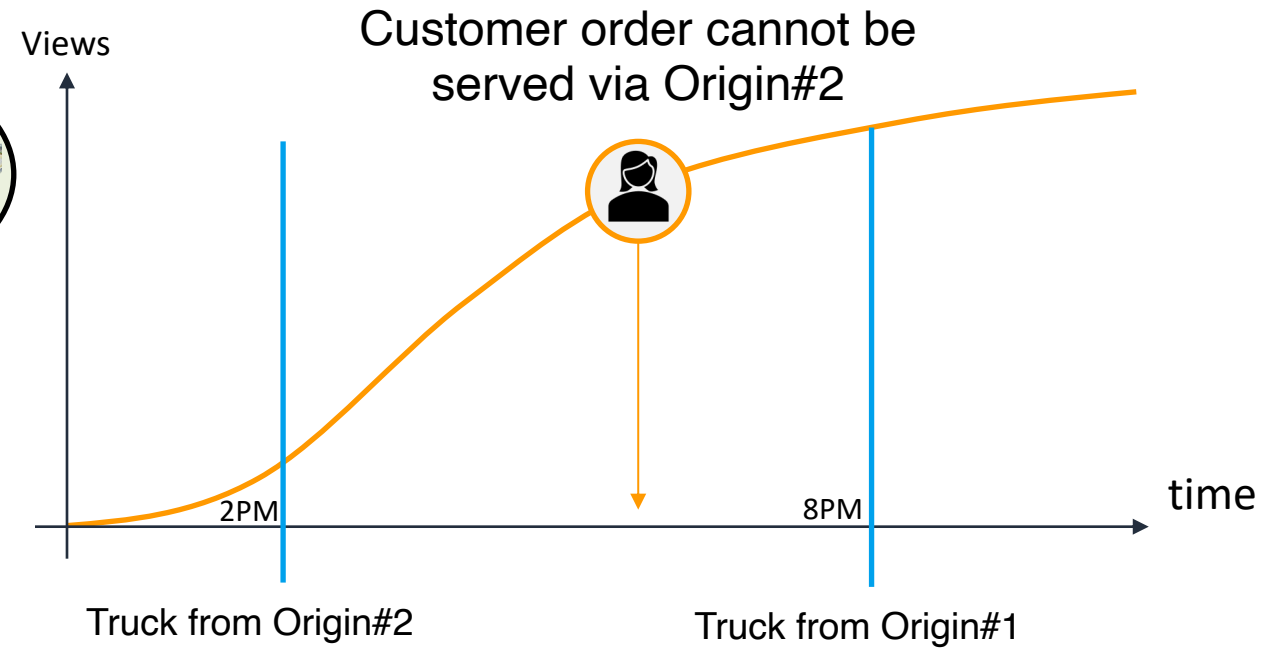
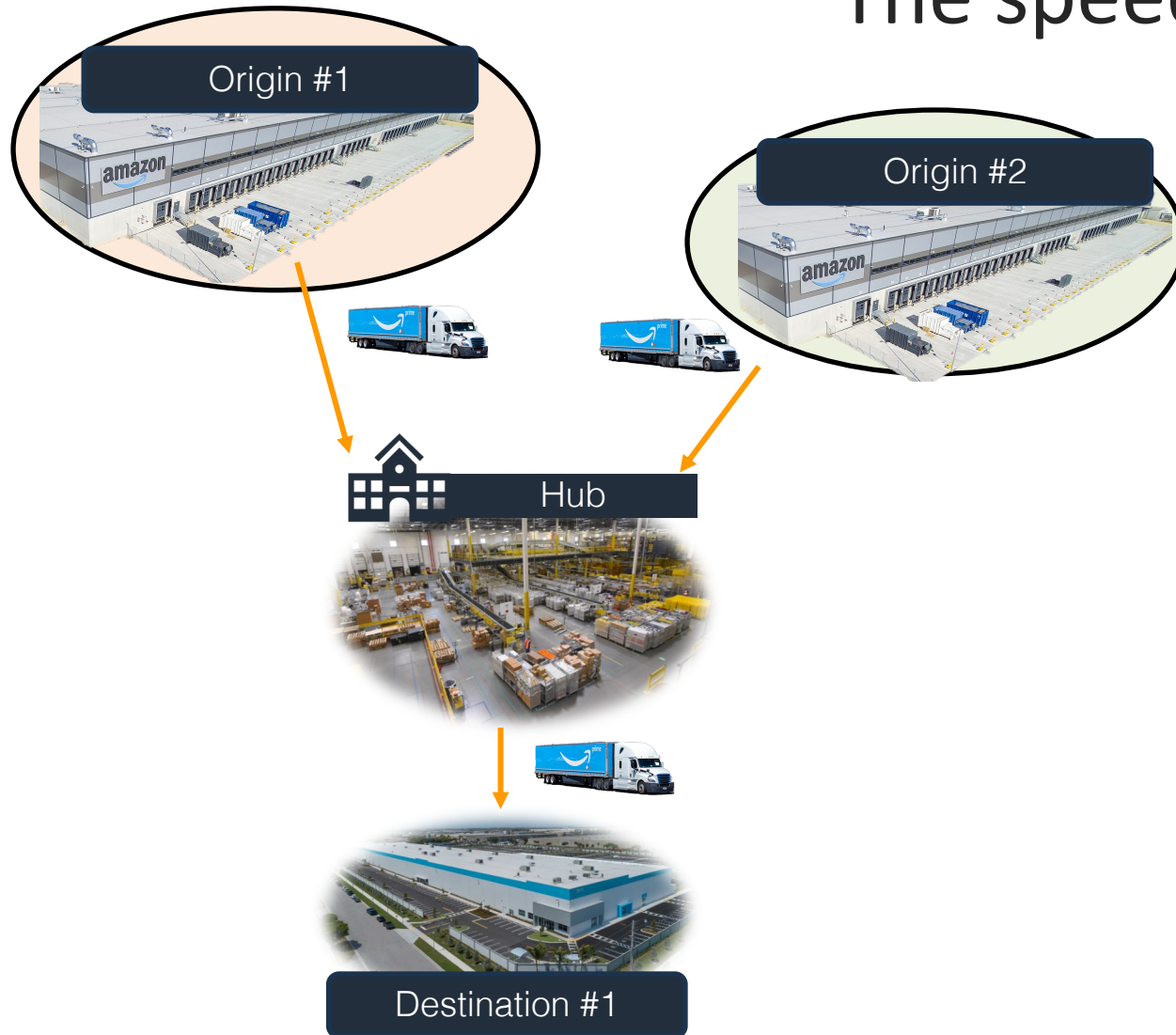
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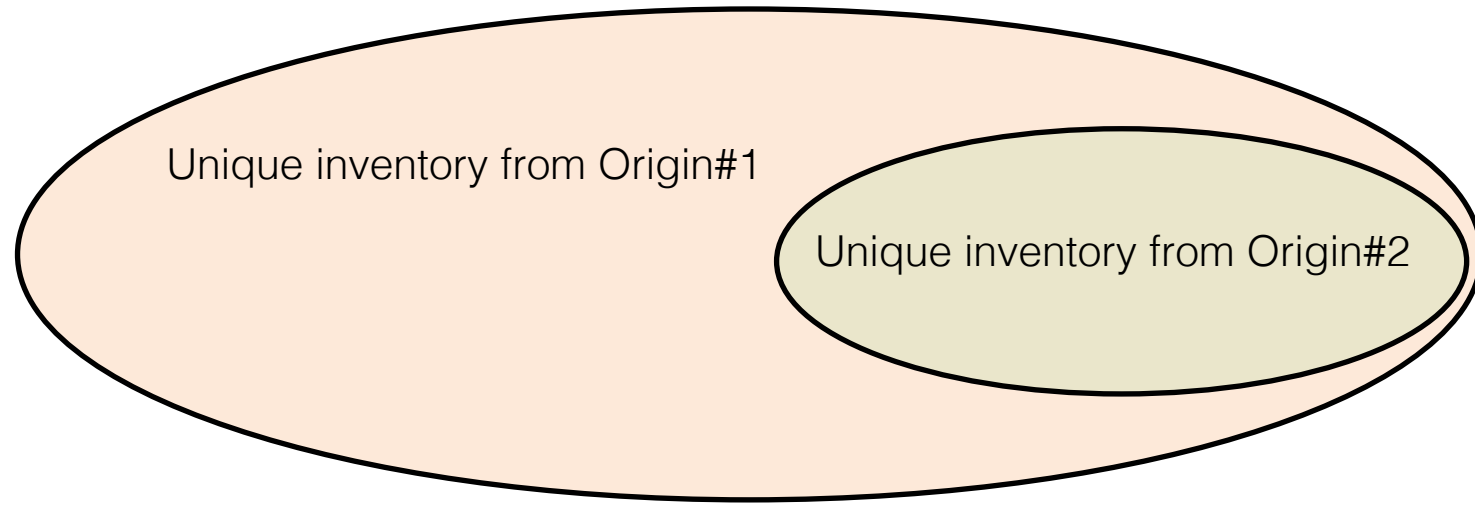
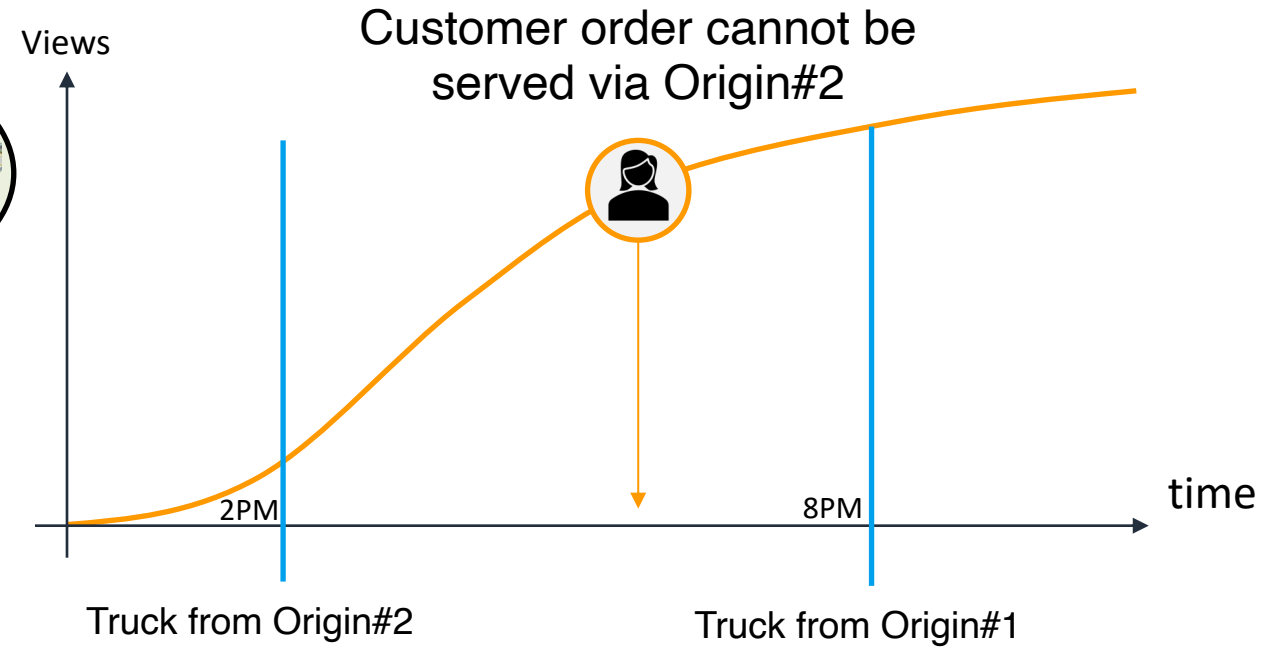
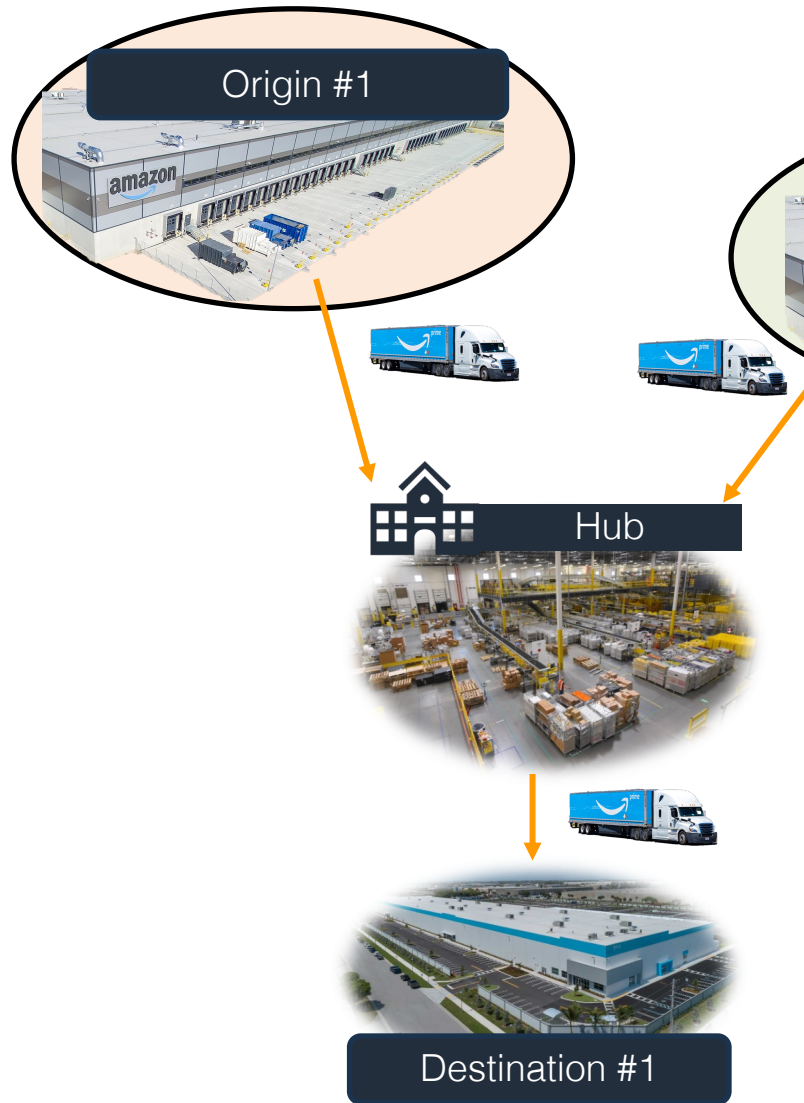
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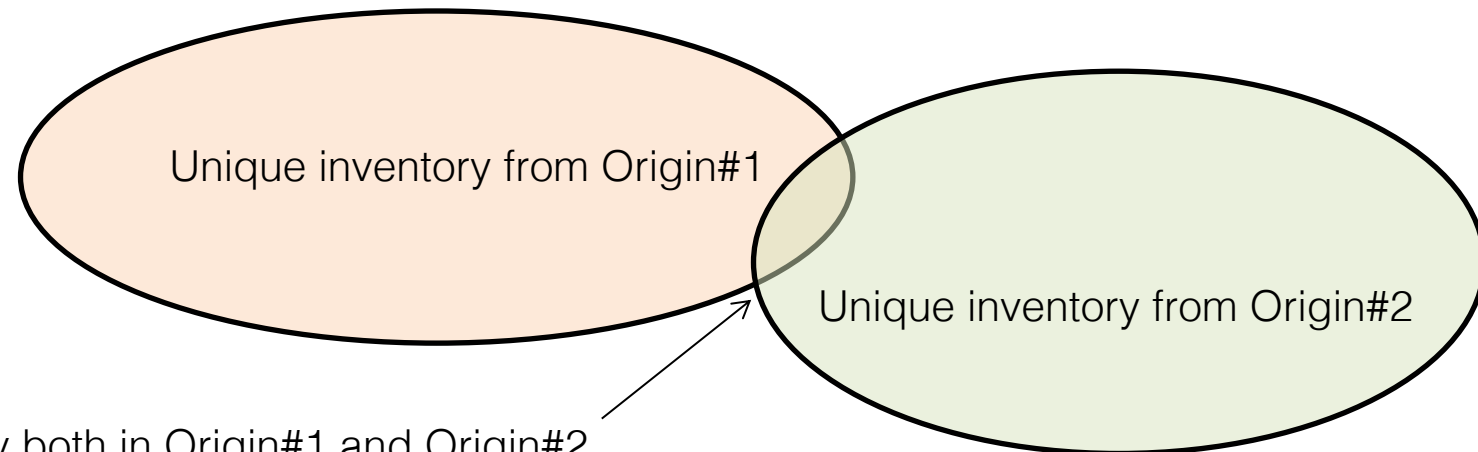
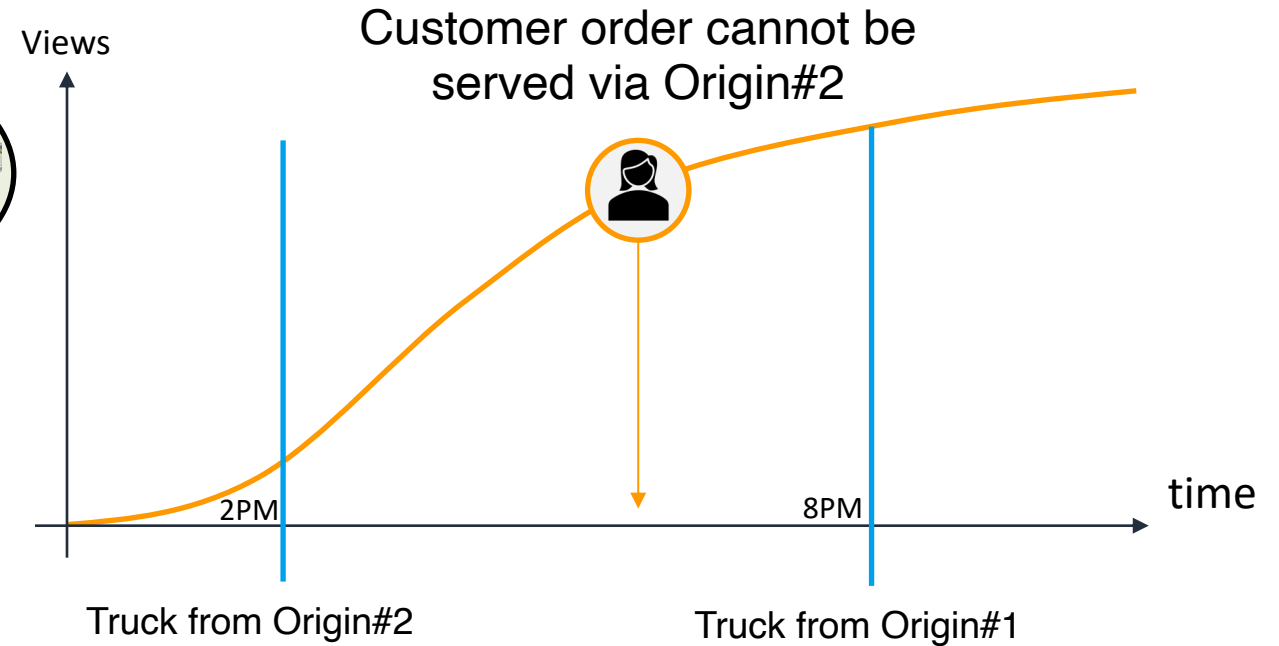
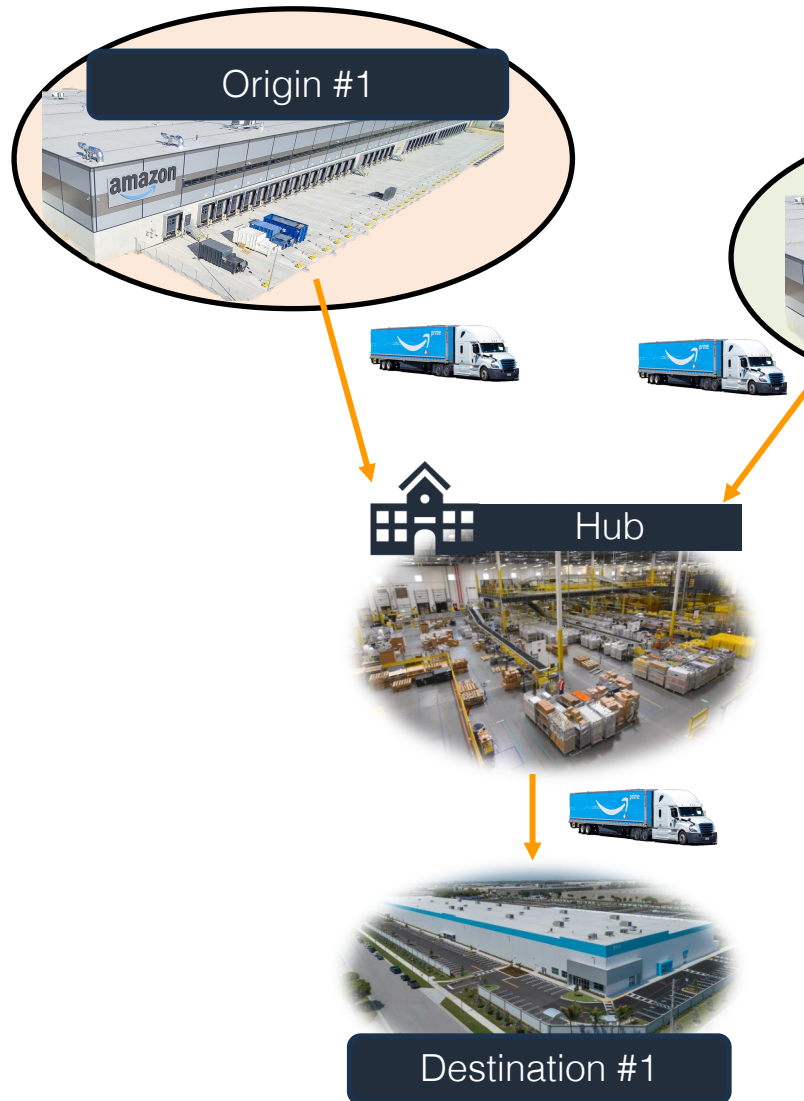
The speed objective



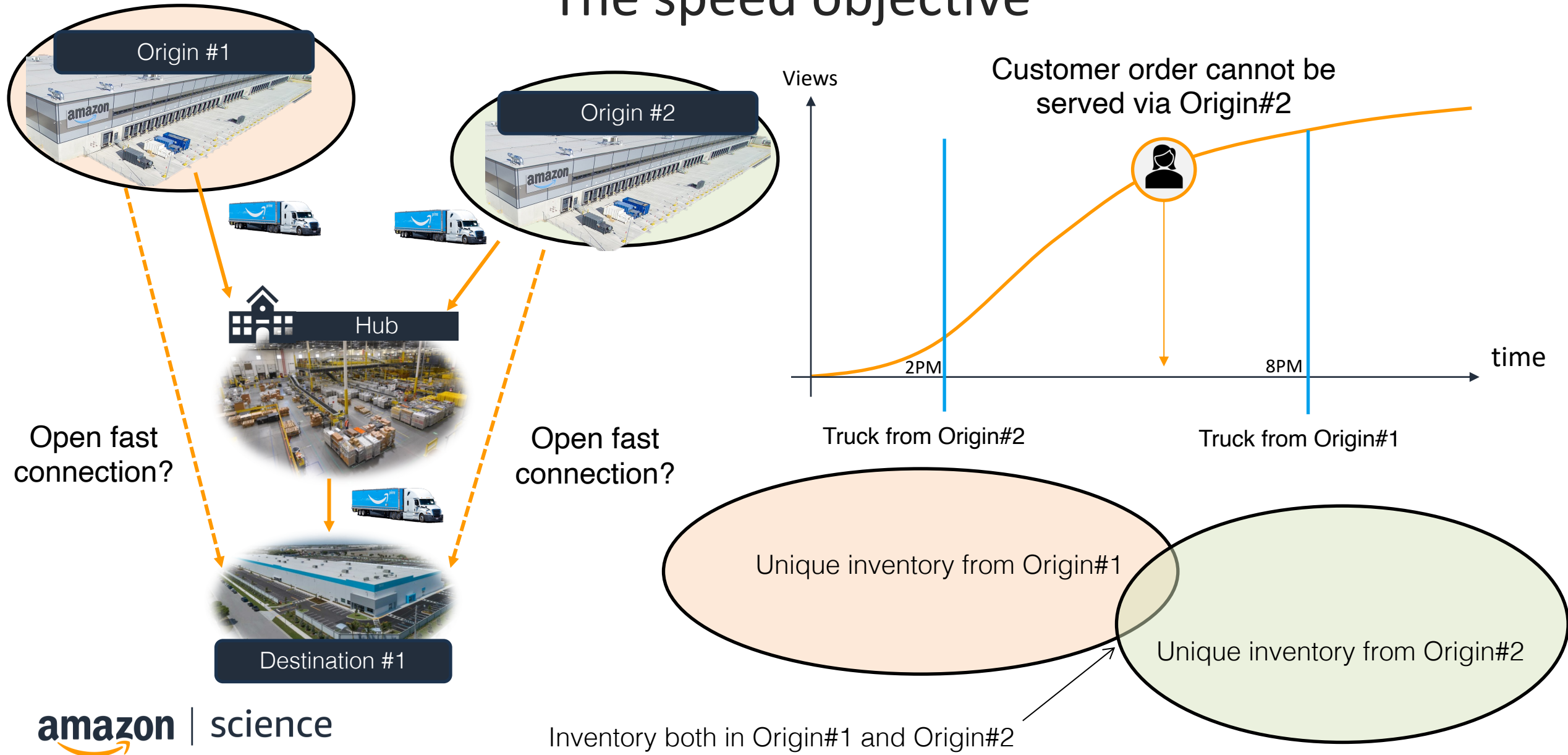
The speed objective



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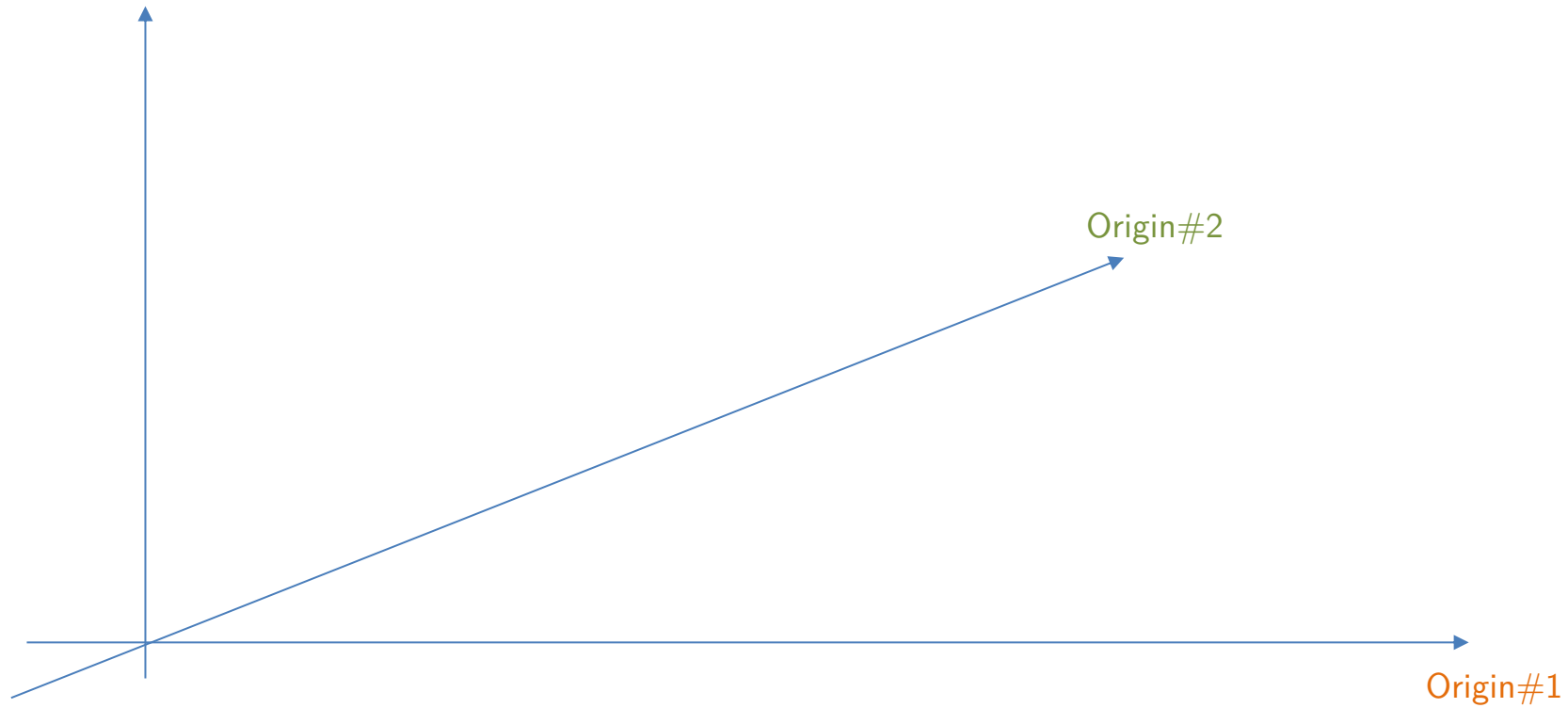


The speed objective



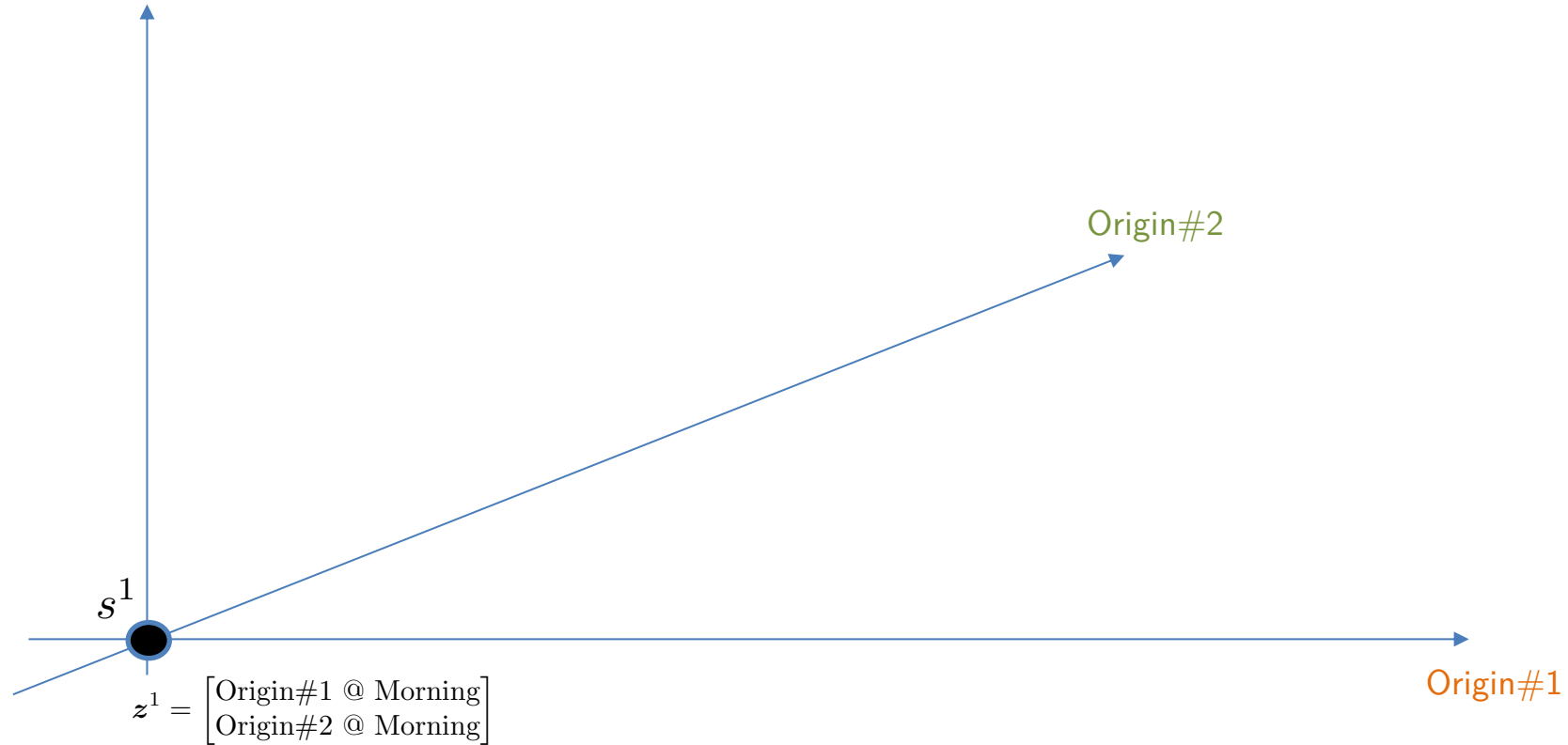
The speed objective

$\text{Speed}(z)$ = unique items that can be delivered in 1-day



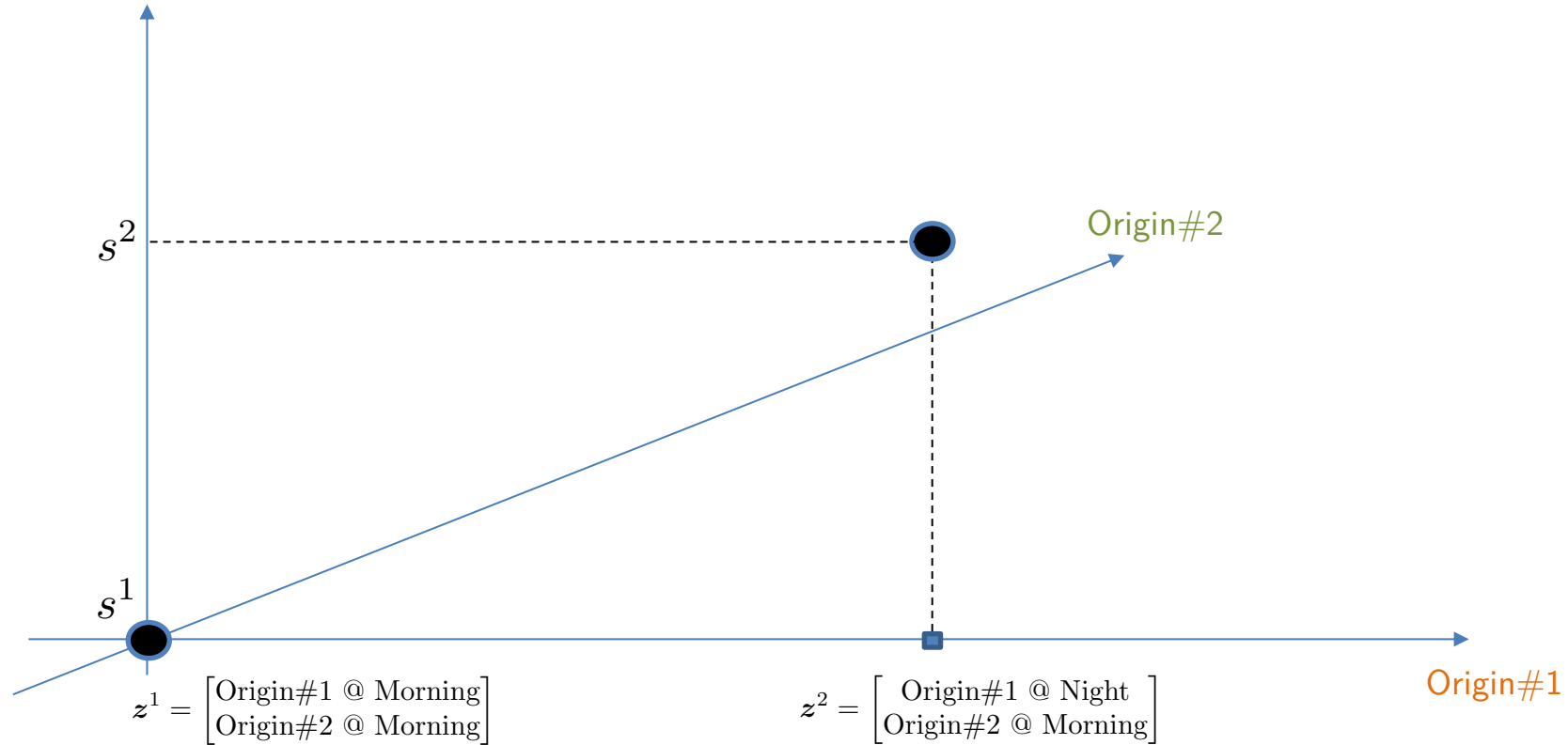
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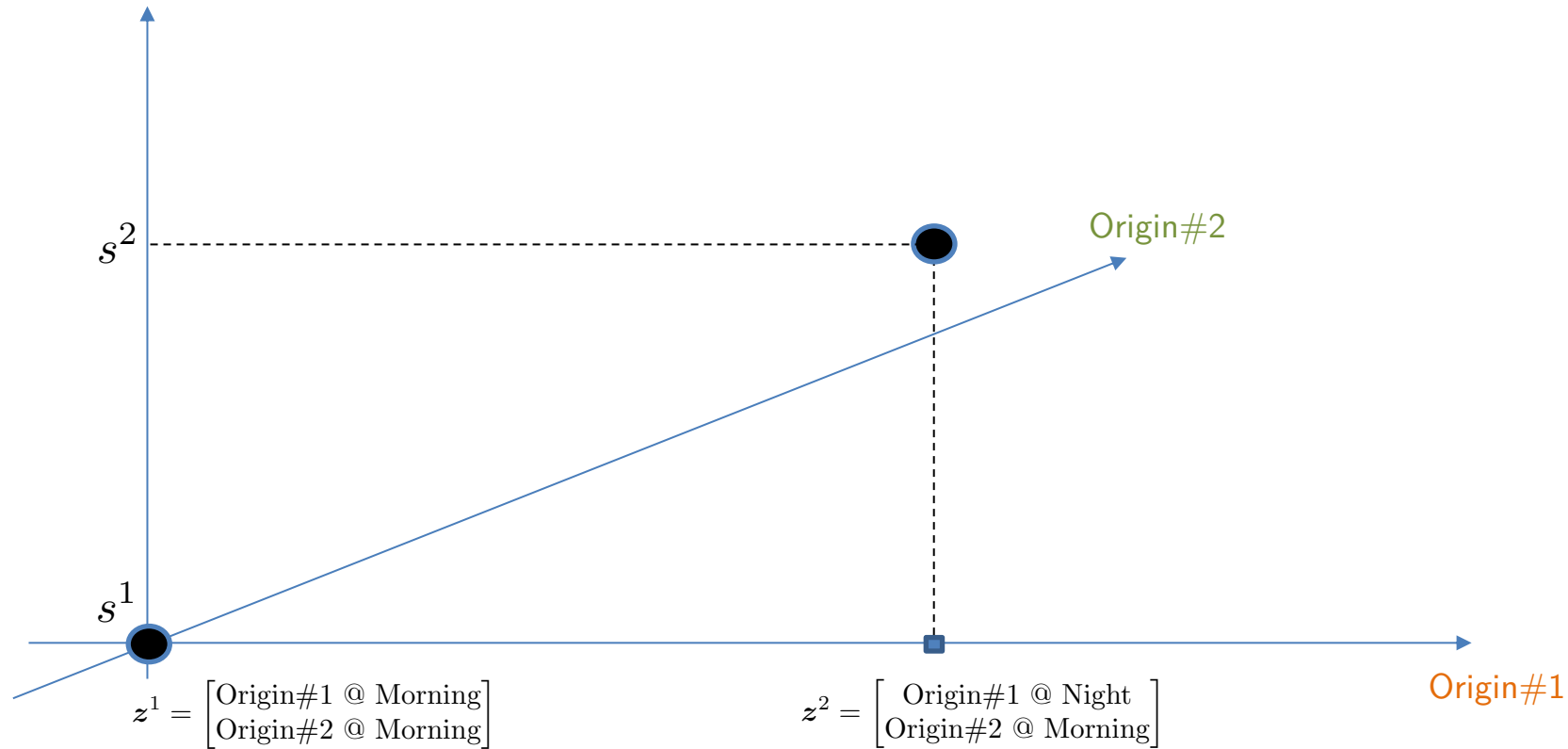
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The speed objective

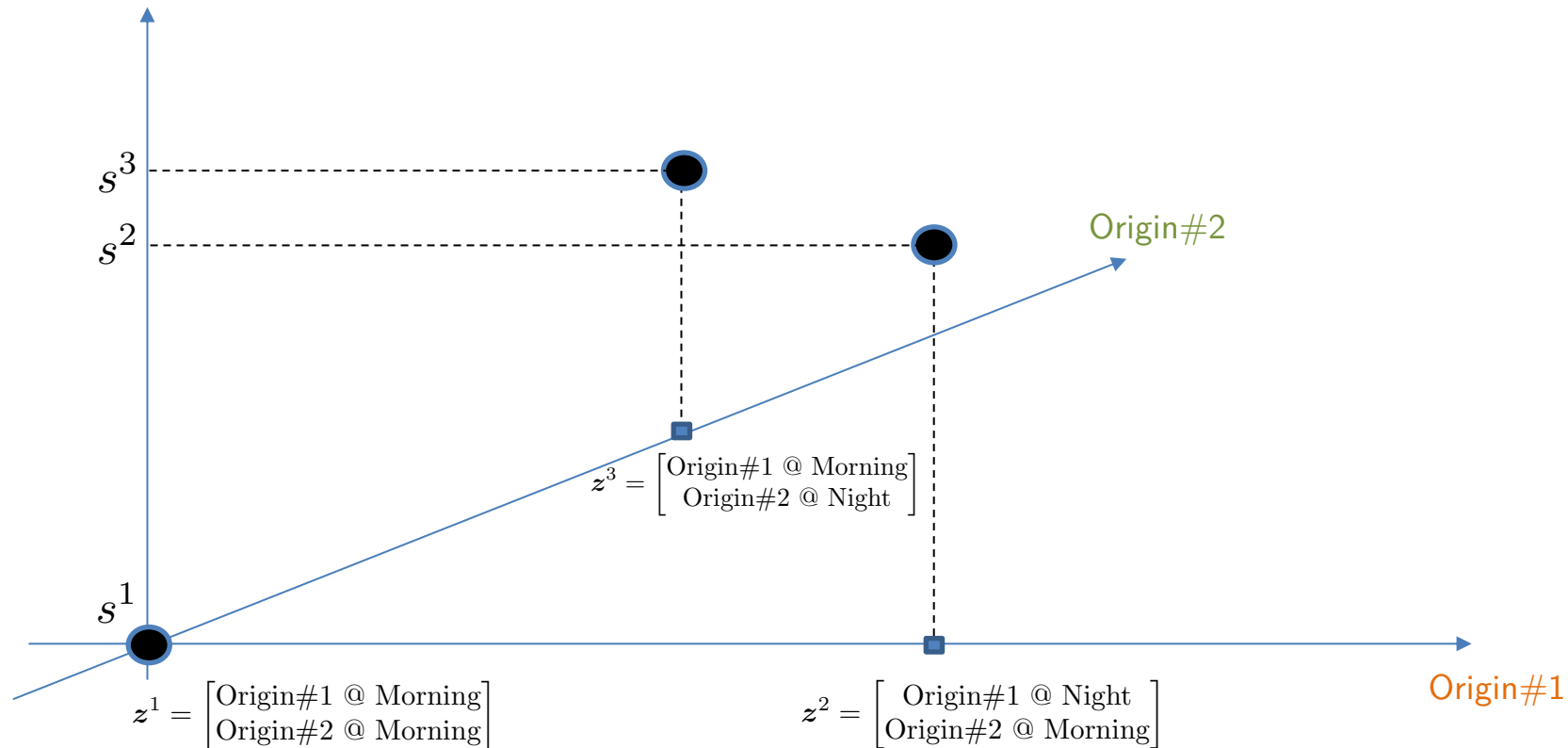
$\text{Speed}(z)$ = unique items that can be delivered in 1-day



$\text{Speed}(z)$ can be computed via a look-up table

The speed objective

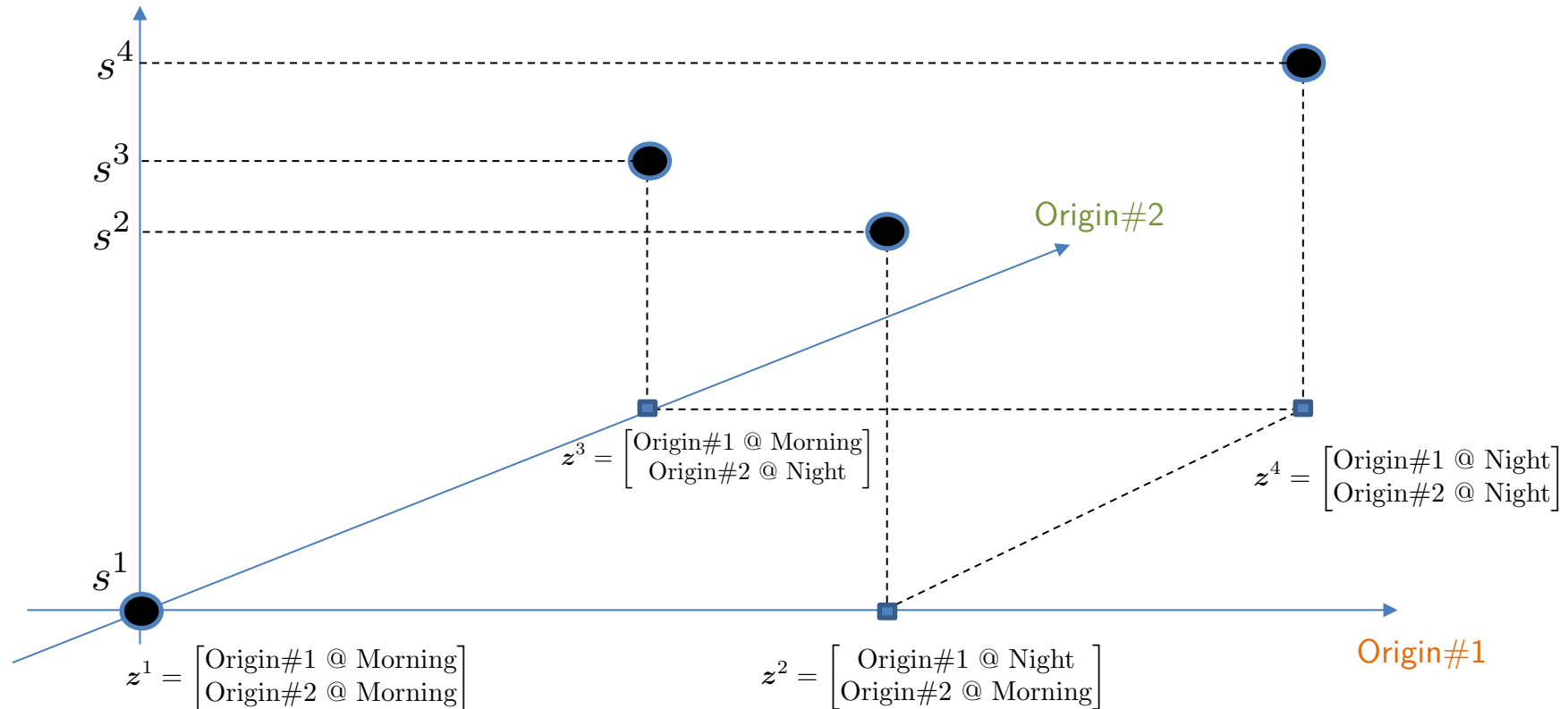
$\text{Speed}(z)$ = unique items that can be delivered in 1-day



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The speed objective

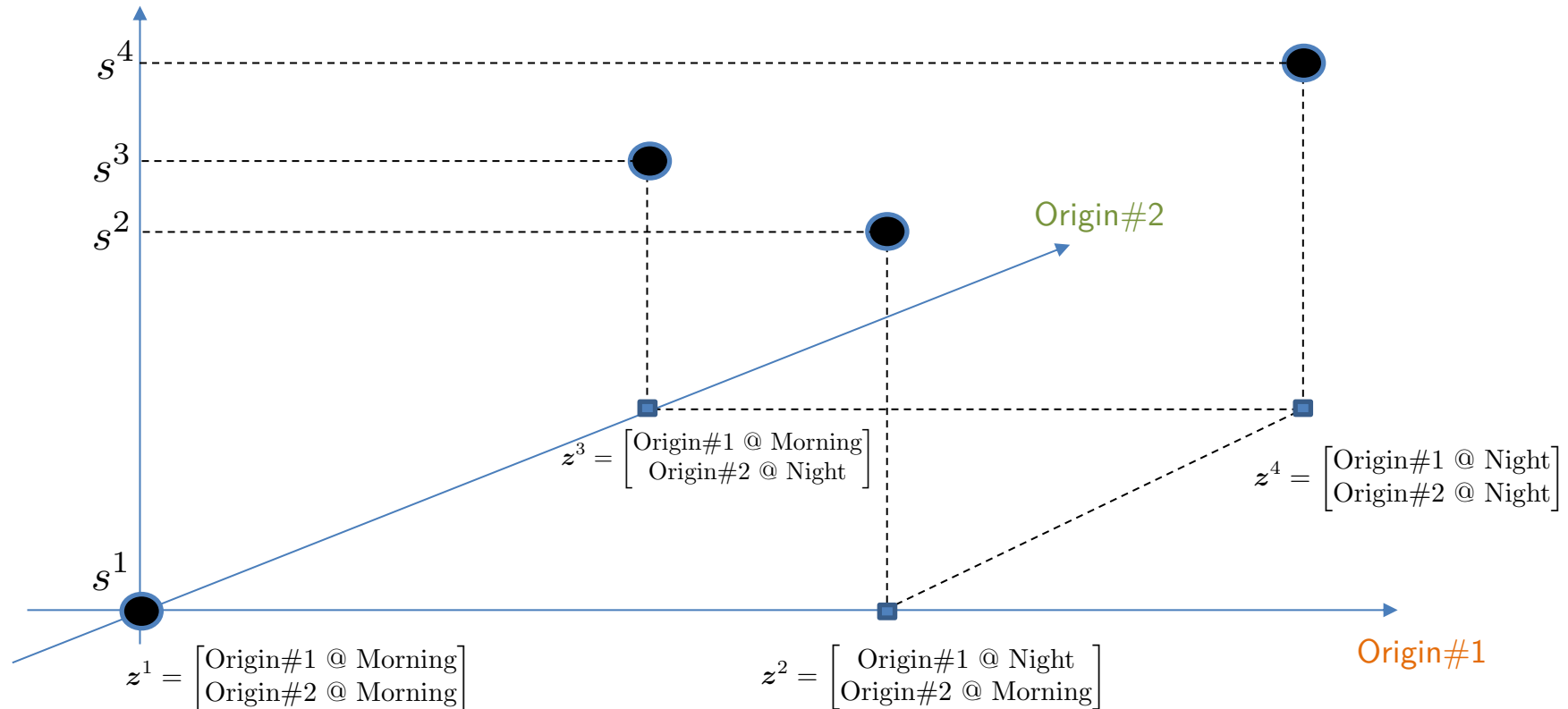
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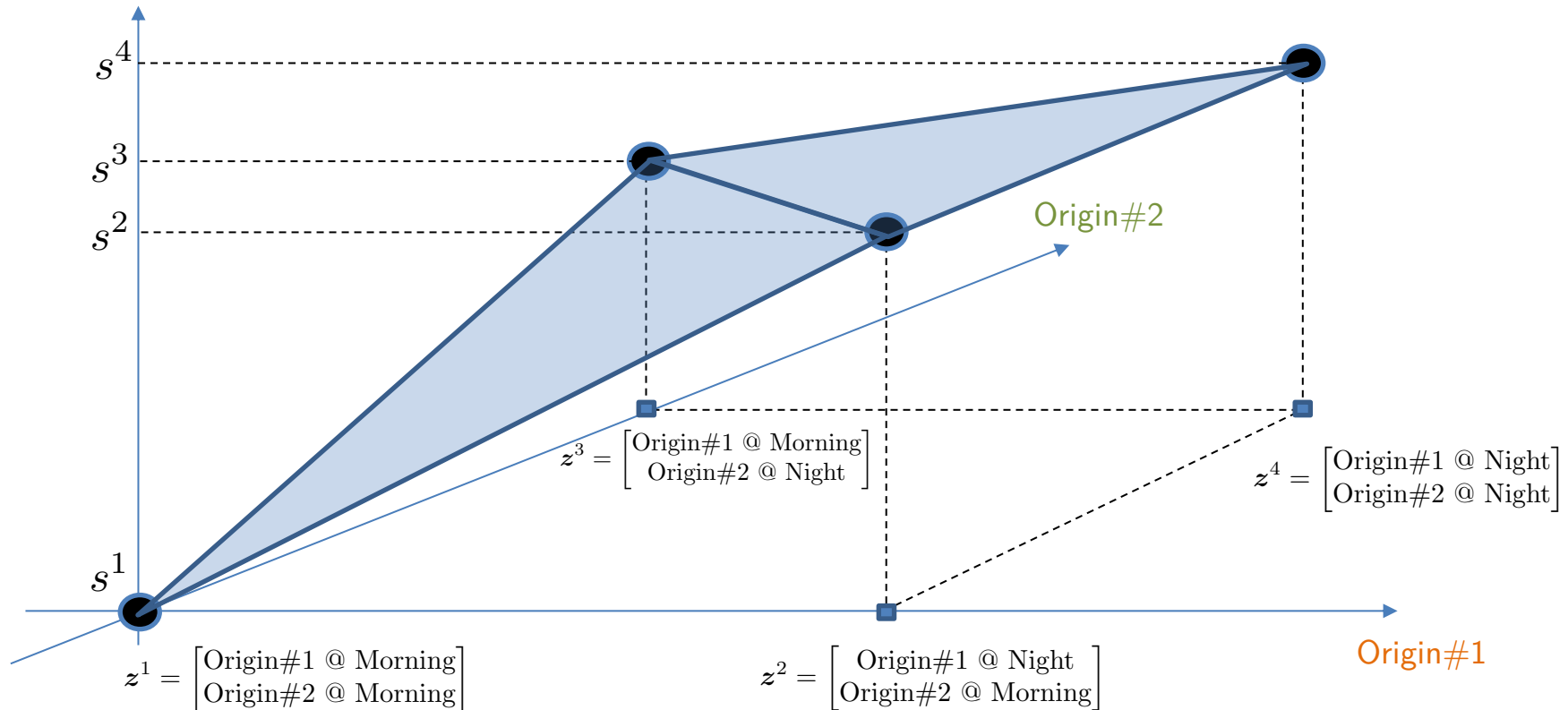
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Approximating the speed objective

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Step 1: Consider truck departure time combinations:

$$z^1 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, z^2 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, z^3 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Night} \end{bmatrix}, z^4 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Night} \end{bmatrix}$$

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Step 2: Compute speed for each combination:

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Approximating the speed objective

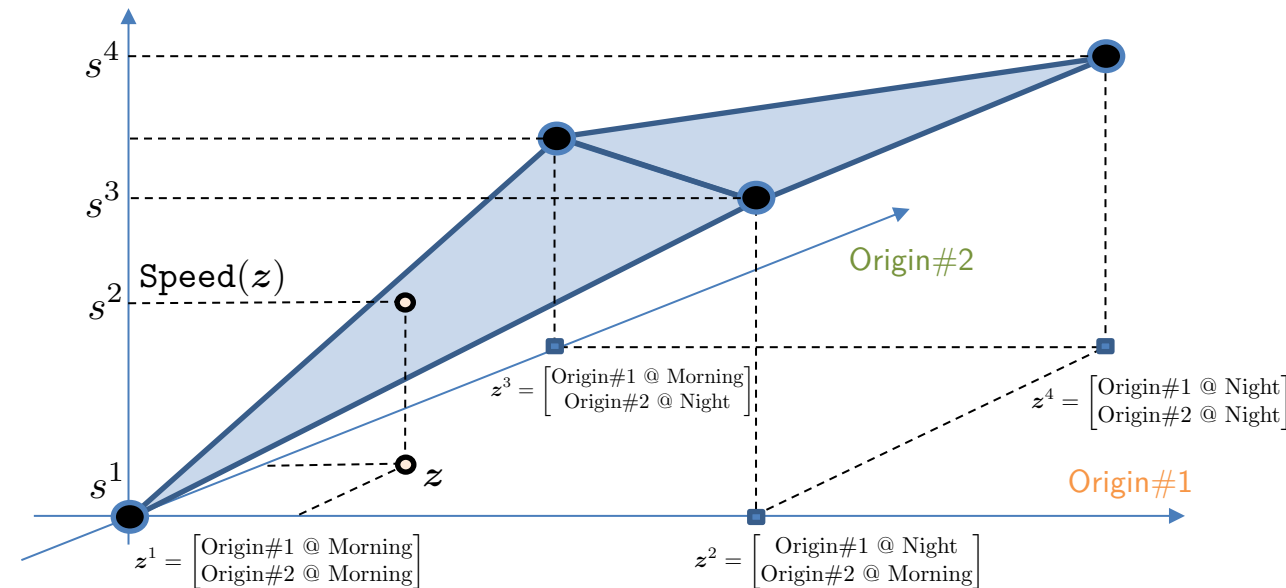
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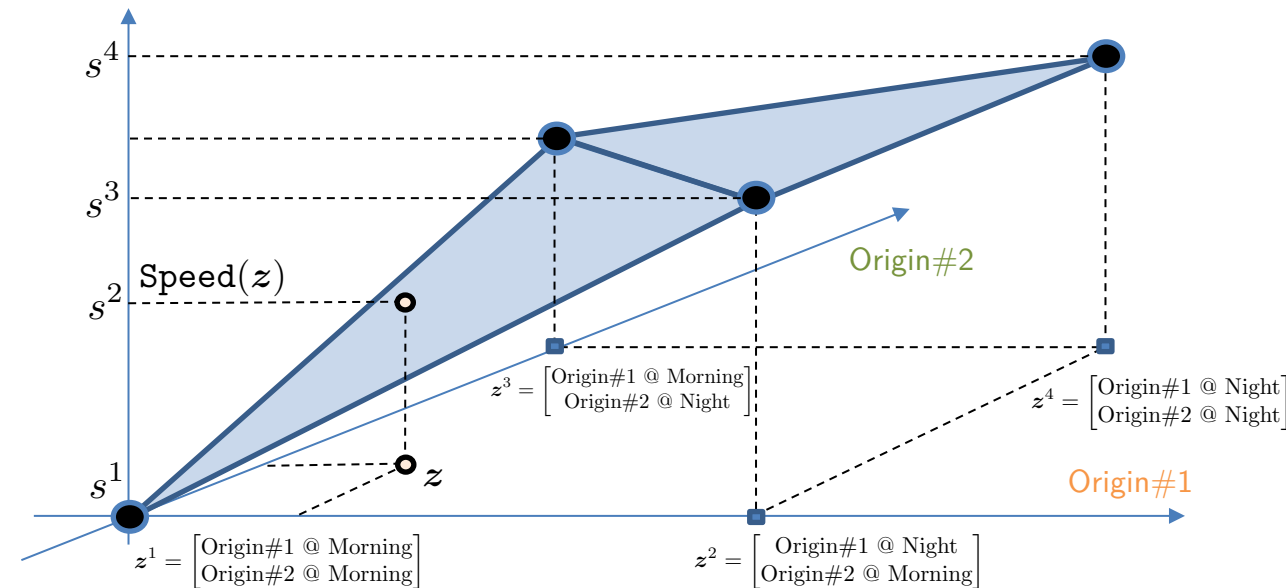
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Step 3: Interpolation via parametric optimization:

$$\begin{aligned} \text{Speed}(z) &= \max_{\alpha \geq 0} \sum_i \alpha^i s^i \\ \text{subject to } &\sum_i \alpha^i z^i = z \\ &\sum_i \alpha^i = 1 \end{aligned}$$



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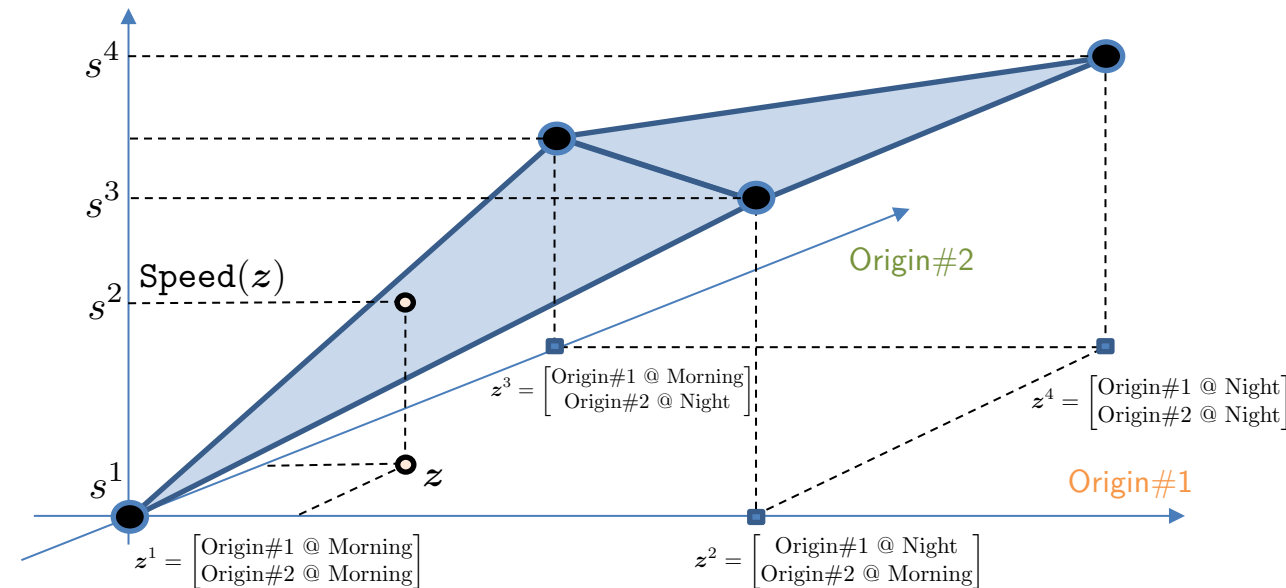
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Parameter defining the optimal value of the LP



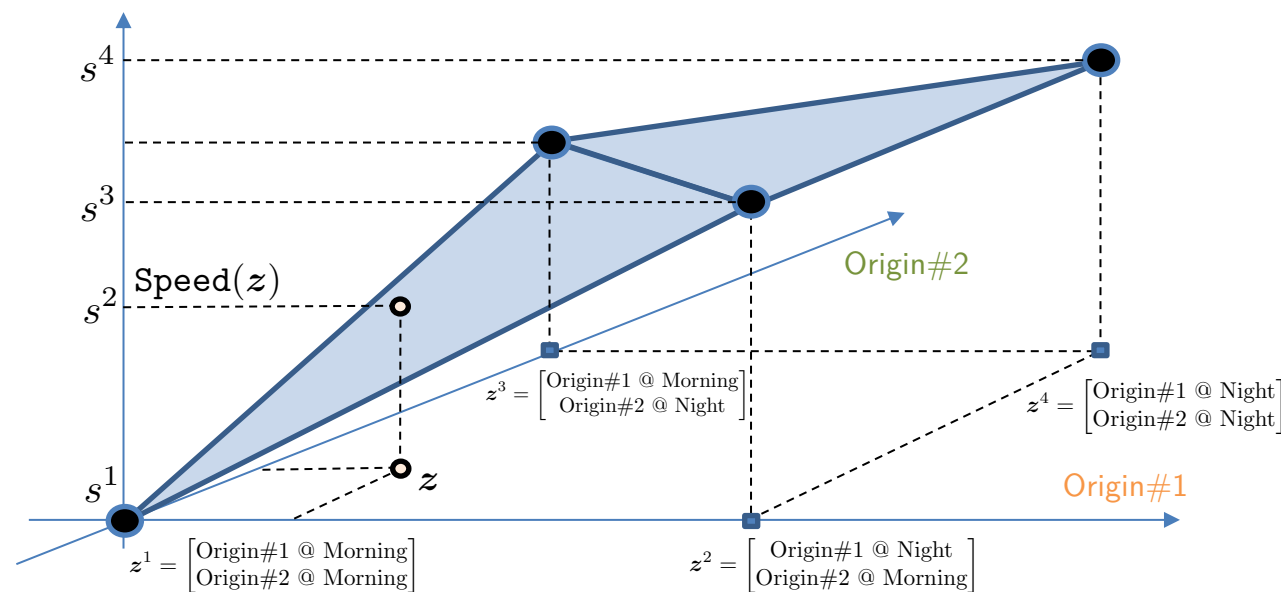
Approximating the speed objective

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Speed(\mathbf{z}) = $\max_{\alpha \geq 0} \sum_i \alpha^i s^i$
 subject to $\sum_i \alpha^i \mathbf{z}^i = \mathbf{z}$
 $\sum_i \alpha^i d^i = 1$

By defining the value of the LP

Parameter defining the
optimal value of the LP



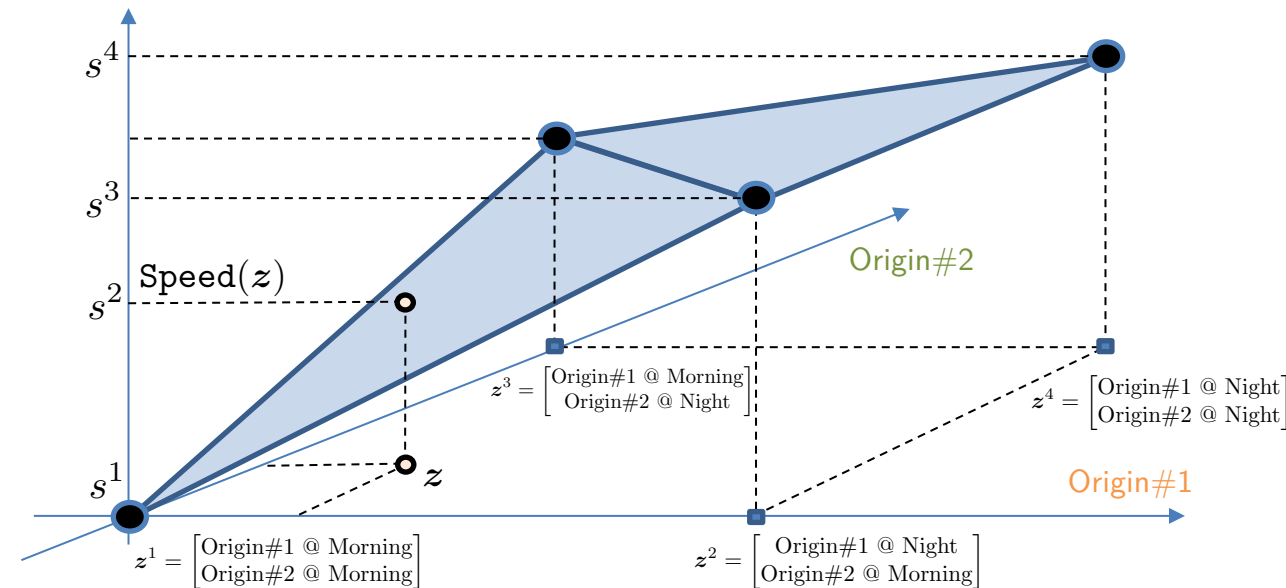
Approximating the speed objective

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{y}, \mathbf{z}} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \text{Speed}(\mathbf{z}) \\ \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \\ & \mathbf{z} \in \text{FeasibleSchedule}(\mathbf{p}) \end{aligned}$$

Step 3: Interpolation via parametric optimization:

$$\begin{aligned} \text{Speed}(\mathbf{z}) = \max_{\alpha \geq 0} \quad & \sum_i \alpha^i s^i \\ \text{subject to} \quad & \sum_i \alpha^i \mathbf{z}^i = \mathbf{z} \\ & \sum_i \alpha^i = 1 \end{aligned}$$

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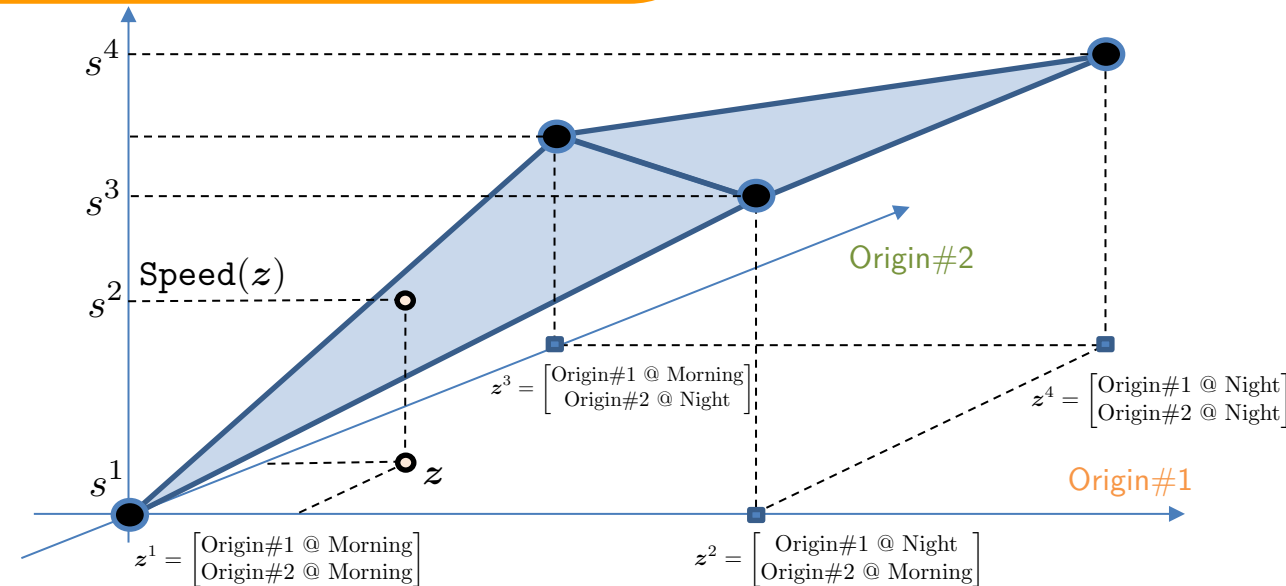
Approximating the speed objective

$$\begin{aligned}
 \min_{\mathbf{p}, \mathbf{y}, \mathbf{z}, \alpha > 0} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \sum_i \alpha^i s^i \\
 \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork}, \\
 & \mathbf{z} \in \text{FeasibleSchedule}(\mathbf{p}) \\
 & \sum_i \alpha^i \mathbf{z}^i = \mathbf{z}, \sum_i \alpha^i = 1
 \end{aligned}$$

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Parameter defining the optimal value of the LP



Outline

- ▶ Motivation
- ▶ Problem formulation
- ▶ Solution strategy
- ▶ Results

Results on a randomly generated datasets

Results on a randomly generated datasets

Results on 4 randomly generated datasets

- ▶ Location of nodes generated at random (cost of operating a truck proportional to distance)
- ▶ Items stored in each warehouse randomly generated
- ▶ Assuming a 0.1 conversion factor from speed and cost

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Results on 4 randomly generated datasets

- ▶ Location of nodes generated at random (cost of operating a truck proportional to distance)
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Speed parametric approximation compute by subsampling datapoint

Algorithm 1: Unique Items Approximation

```
1 Inputs:  $\kappa, \mathcal{D}, \mathcal{F}$ ;  
2 for  $d \in \mathcal{D}$  do  
3   Initialize  $\Lambda_d = \emptyset$ ;  
4   Sort by number of unique item the FCs that can offer 1-day delivery to DS  $d$ ;  
5   Compute the set  $\mathcal{Z}_d$  of all possible combinations of speed lane assignments for top  $\kappa$  FCs;  
6   For all  $z_d^i \in \mathcal{Z}_d$ , add the tuple  $(z_d^i, U_d(z_d^i))$  to the set  $\Lambda_d$ ;  
7   Compute the speed lane assignments from (12) for remaining  $(n_f - \kappa)$  FCs;  
8   For all  $\tilde{z}_d^i \in \tilde{\mathcal{Z}}_d$ , add the tuple  $(\tilde{z}_d^i, U_d(\tilde{z}_d^i))$  to the set  $\Lambda_d$ ;  
9 Return: Set of vectors and cost coefficients  $\Lambda_d$  for  $d \in \mathcal{D}$ .
```

Take all combination for top κ origins

Results on a randomly generated datasets

Baseline
method only
considering cost

n_F	n_D	Costs	App. Rev.	Rev.	Cost - (App. Rev.)	Cost - Rev.	κ	%Gap
10	10	689.1	-	342.8	689.1	346.3	-	0
10	10	698.5	383.7	383.7	314.8	314.8	1	0
10	10	698.5	383.7	383.7	314.8	314.8	5	0
10	10	698.5	383.7	383.7	314.8	314.8	10	0
10	20	1419.2	-	370.7	1419.2	677.7	-	1.8
10	20	1459.7	405.9	407	647.9	645.7	1	3.4
10	20	1434.1	397.8	400.7	638.5	632.7	5	2.8
10	20	1435.5	403.5	403.5	628.4	628.4	10	2.5
20	10	1392.2	-	770.4	1392.2	621.8	-	0.1
20	10	1422.9	807.3	815.7	615.6	607.2	1	0.1
20	10	1422.9	807.3	815.7	615.6	607.2	5	0.1
20	10	1413.6	806.5	811.2	607.1	602.4	10	0.1
50	10	3375.8	-	1845.2	3375.8	1530.6	-	1.8
50	10	3528.2	2010	2032.2	1518.2	1496.0	1	7.5
50	10	3499.0	2031.5	2039.1	1467.5	1459.9	5	3.8
50	10	3489.8	2025.3	2037.3	1464.4	1452.4	10	3.6
100	100	62204.2	-	3651.1	62204.2	25693.2	-	2.8
100	100	65911.4	4159.3	4175.7	24317.9	24154.2	1	11.2
100	100	65531.7	4156.8	4173.1	23963.2	23800.5	5	9.5
100	100	65258.6	4154.3	4172.1	23714.9	23537.2	10	8.4

8.36%
improvement
over baseline

Summary

Problem

- ▶ Speed and cost should be jointly optimised
- ▶ Speed objective is submodular and can be evaluated with a data query

Solution

- ▶ Leverage parametric optimization
- ▶ Subsampling strategy to reduce complexity

Results

- ▶ 8.36% speed and cost benefits compared to baseline
- ▶ No additional computational cost compared to baseline

