Speed-aware network design

A parametric optimisation approach

Ugo Rosolia, Marc Bataillou Almagro, George Iosifidis, Amit Kumar, Georgios Paschos

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Outline

- Motivation
- Problem formulation
- Solution strategy
- Results

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Middle Mile Network Design

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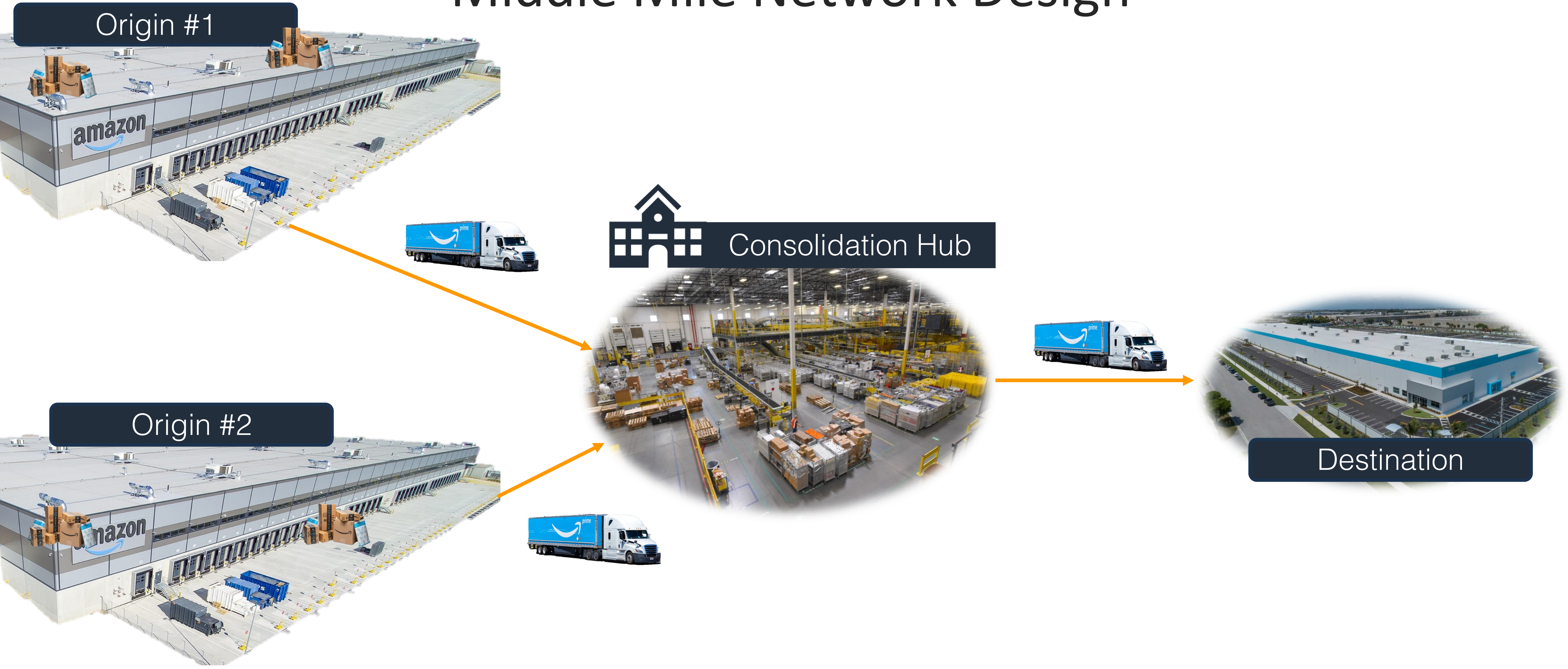

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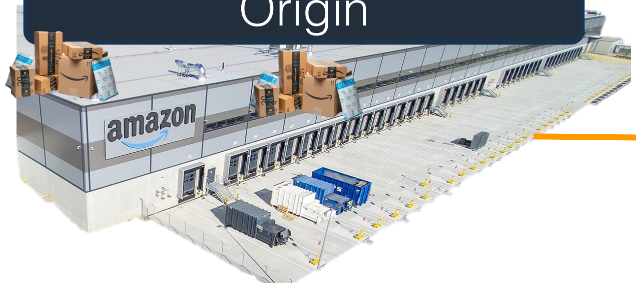
Middle Mile Network Design

Origin



Consolidation Hub

Destination

Middle Mile Network Design

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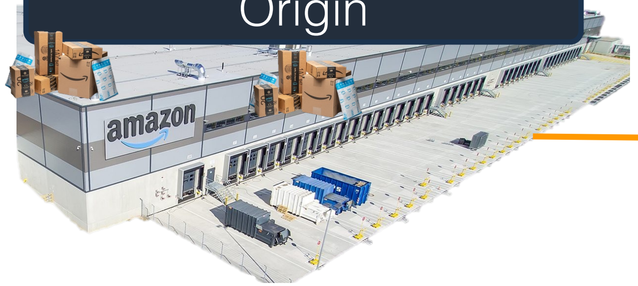

Consolidation Hub

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Key decisions

1. Connectivity, i.e., buildings to connect.
2. Timing, i.e., trucks departure times.

Middle Mile Network Design

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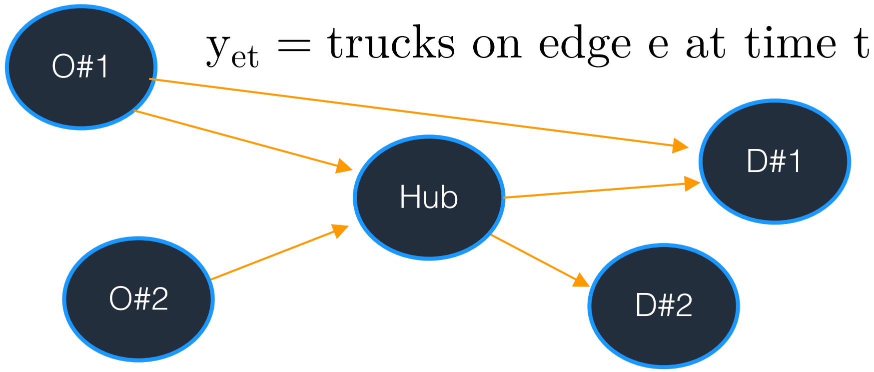

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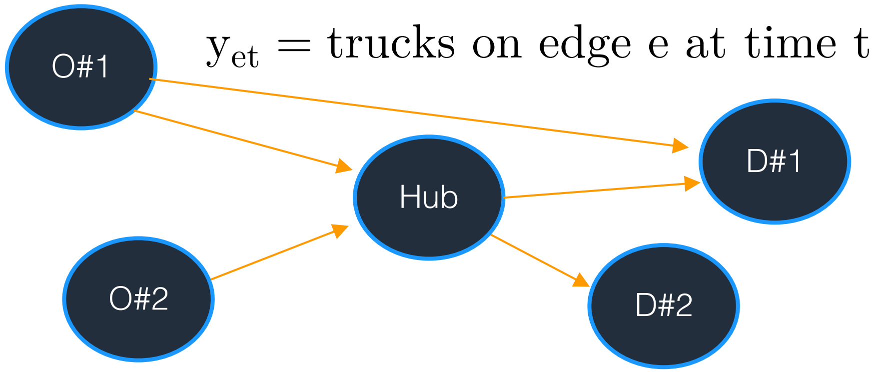

Intractable at Amazon's scale!

Why considering speed?

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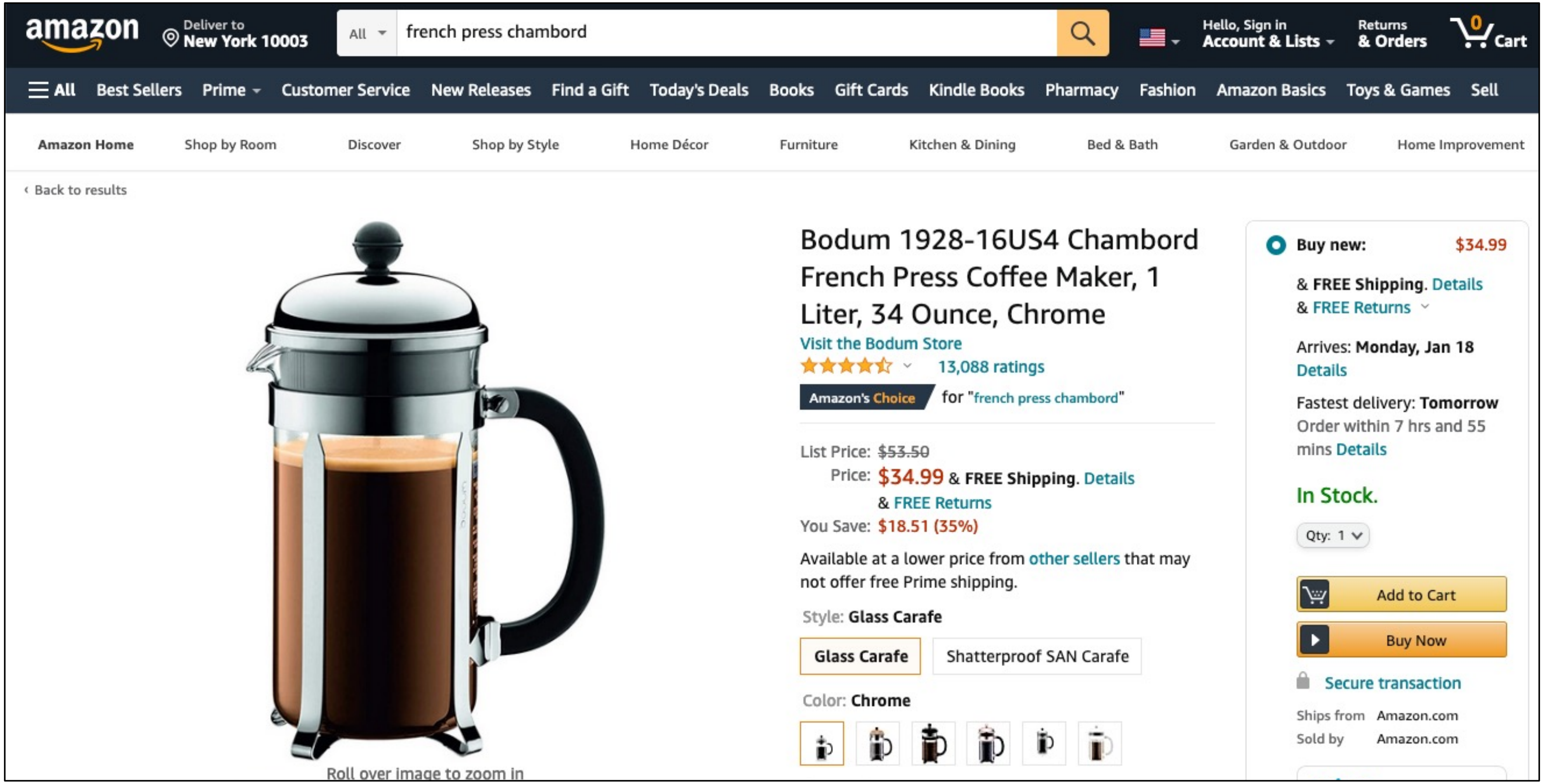

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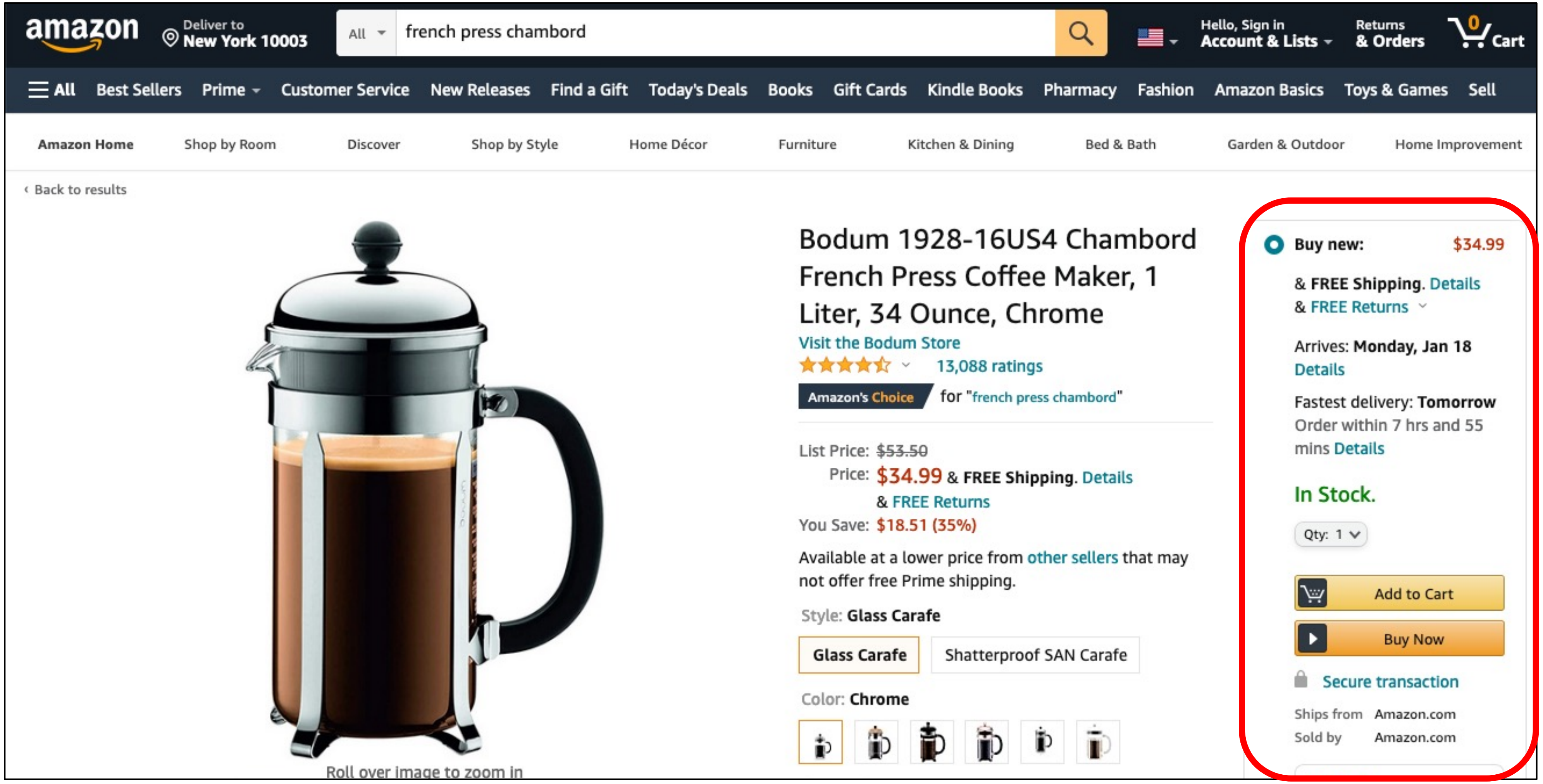

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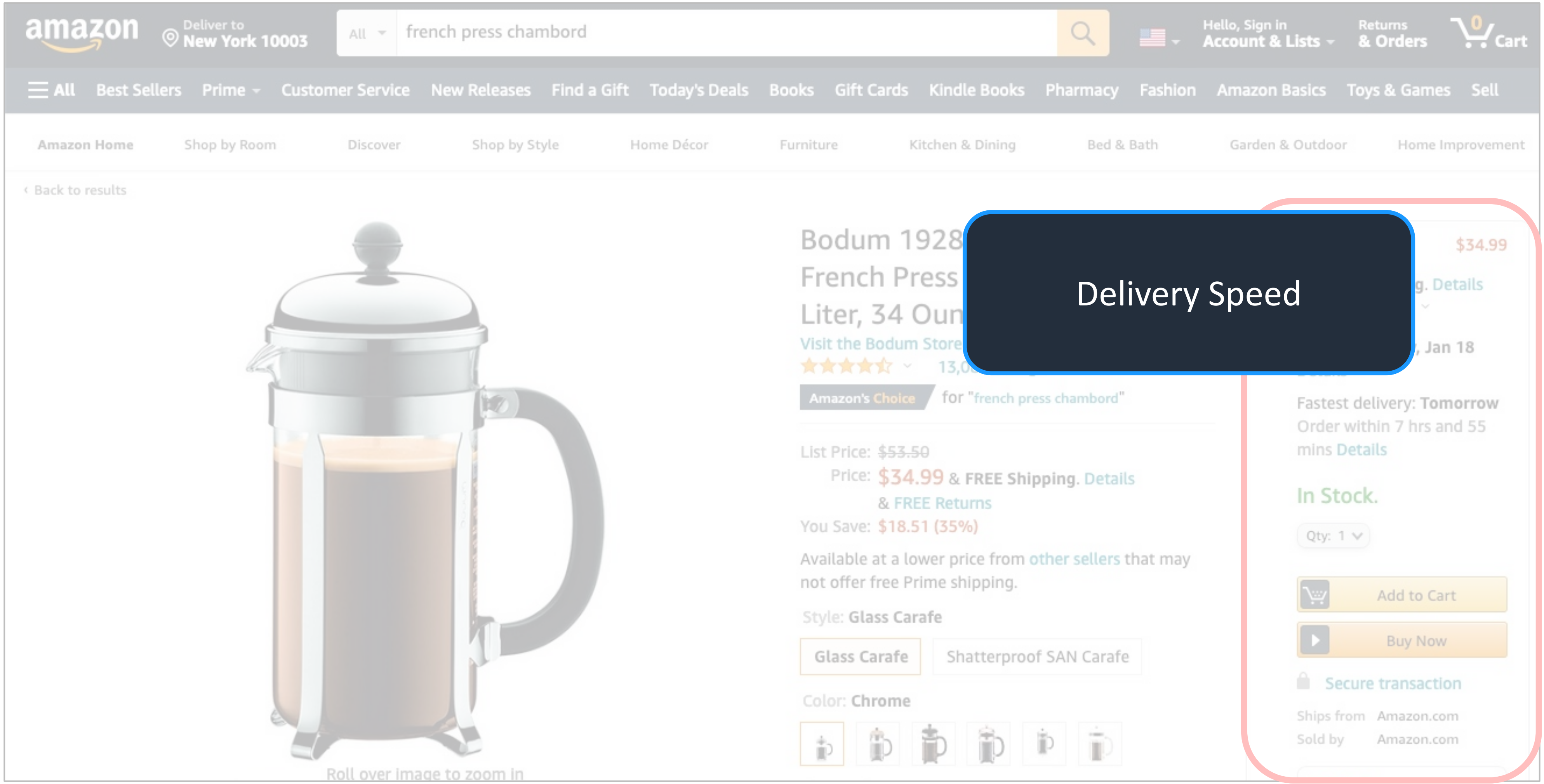

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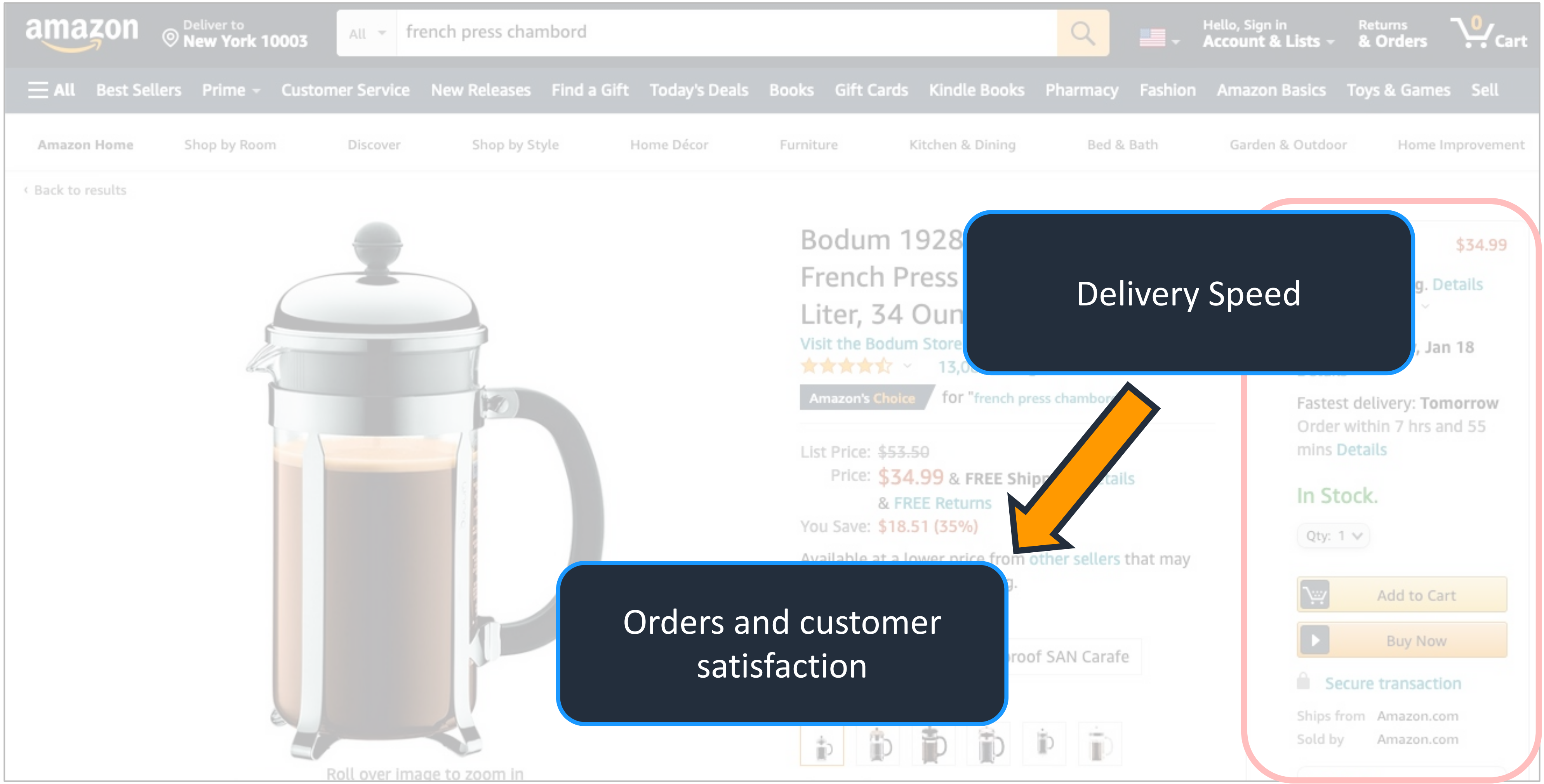

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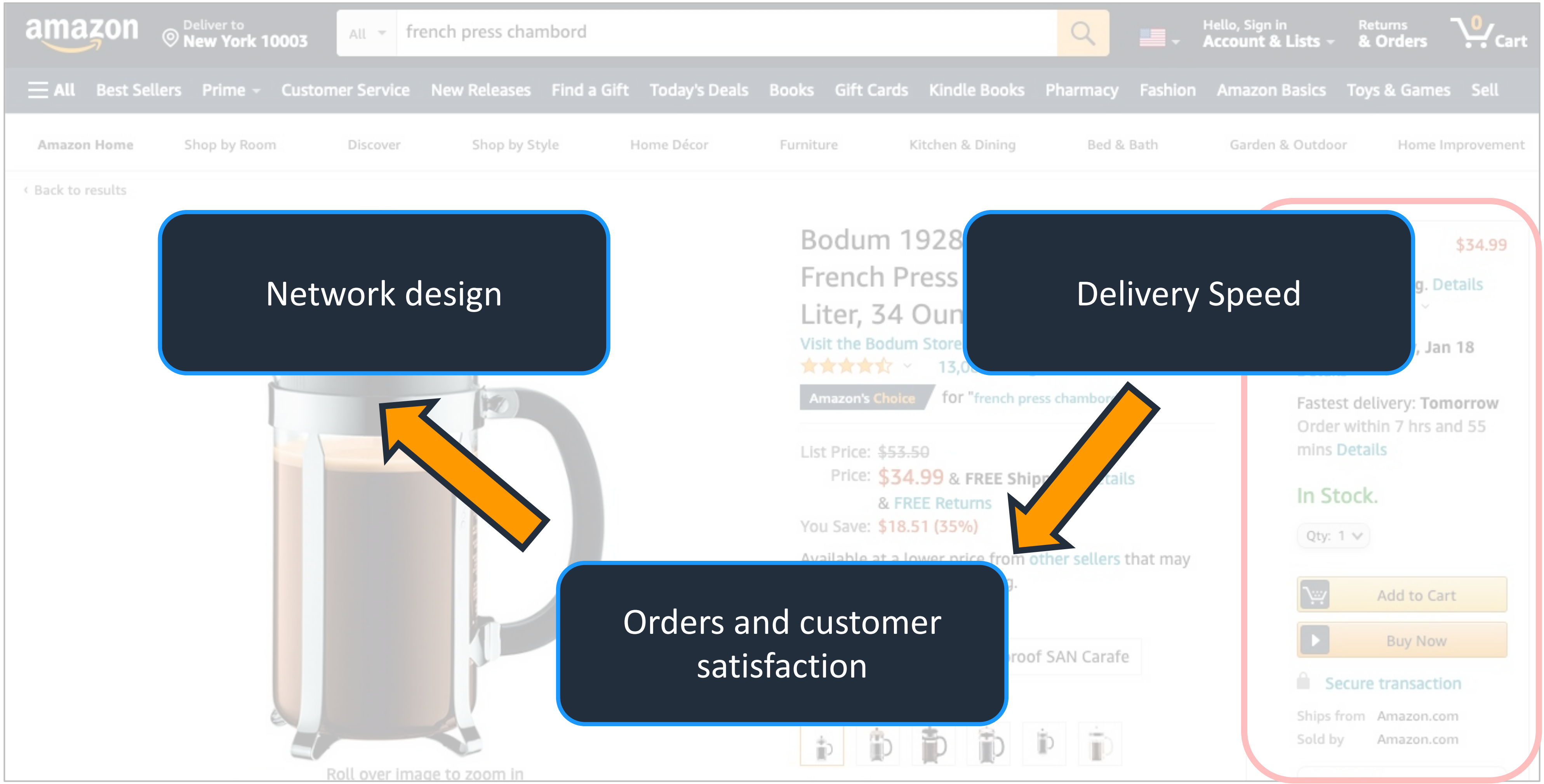

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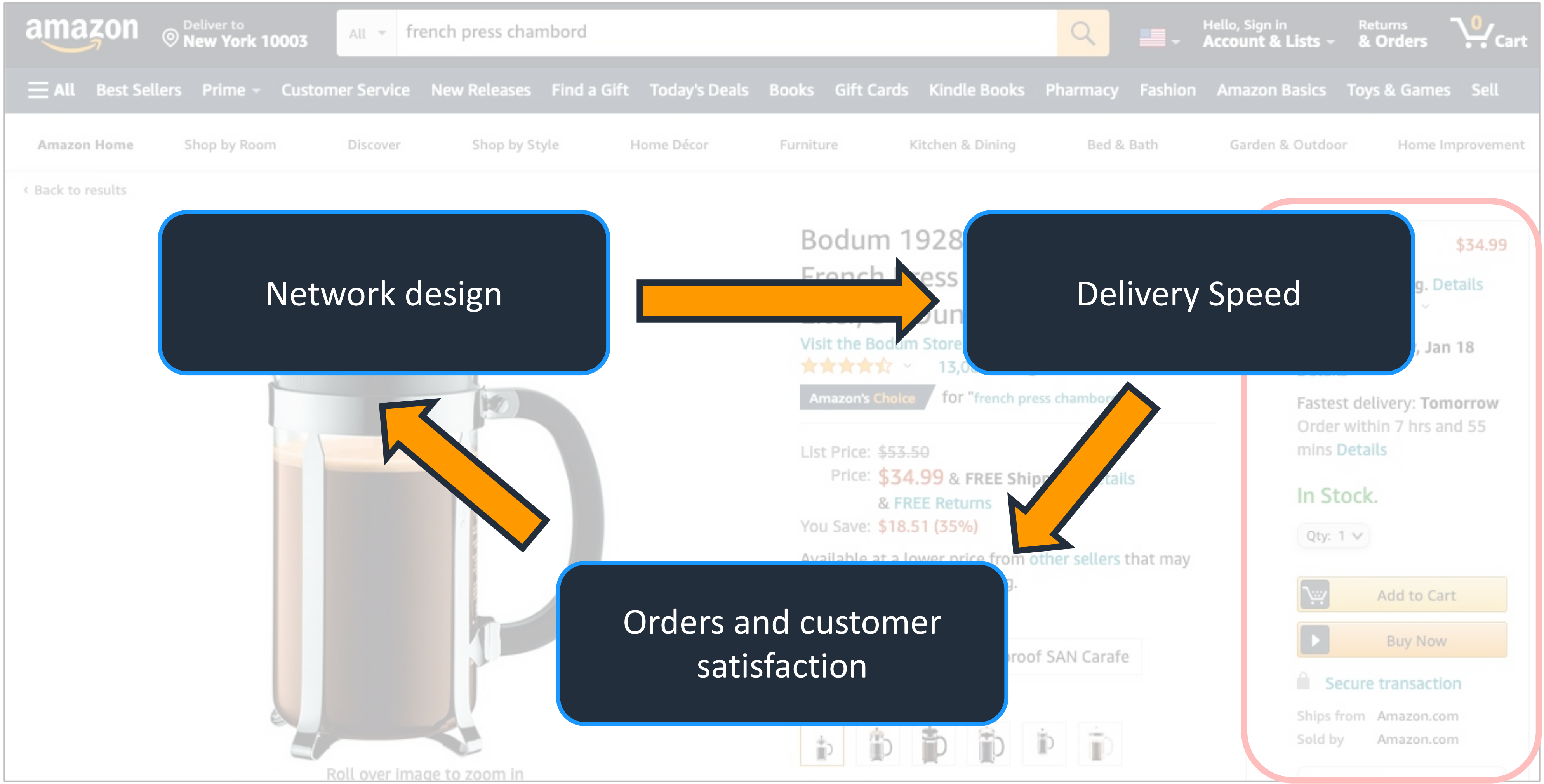

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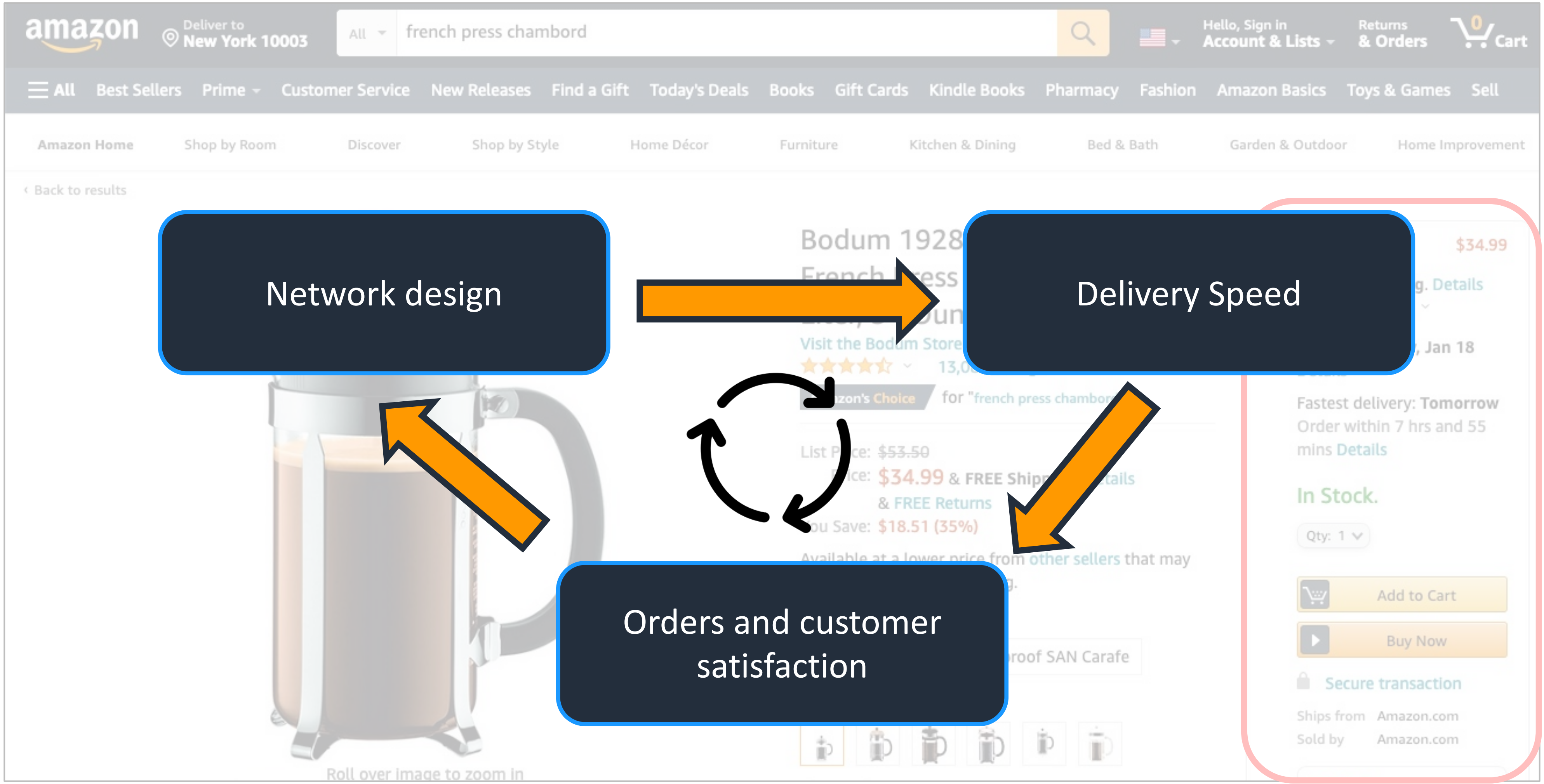
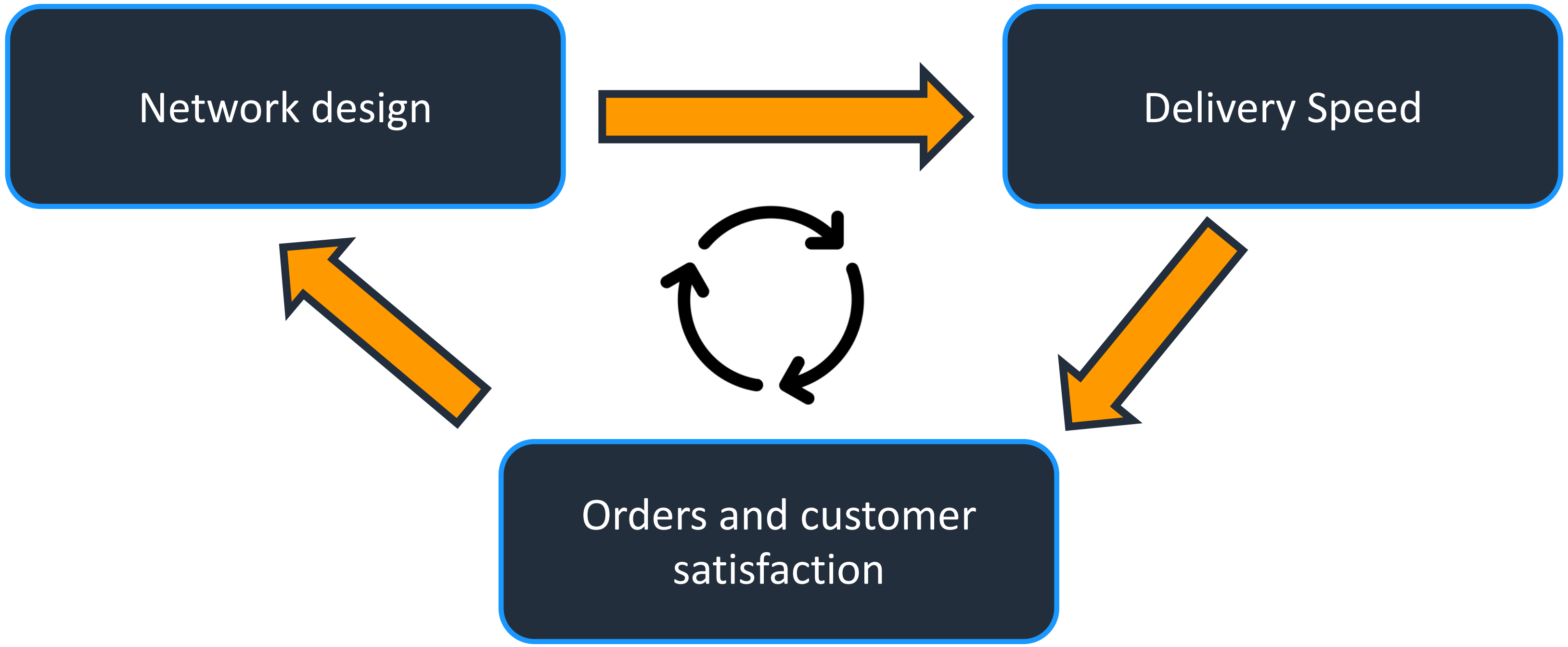


Why considering speed?

Key objective: Design the <u>cheapest and fastest</u> network for our customers



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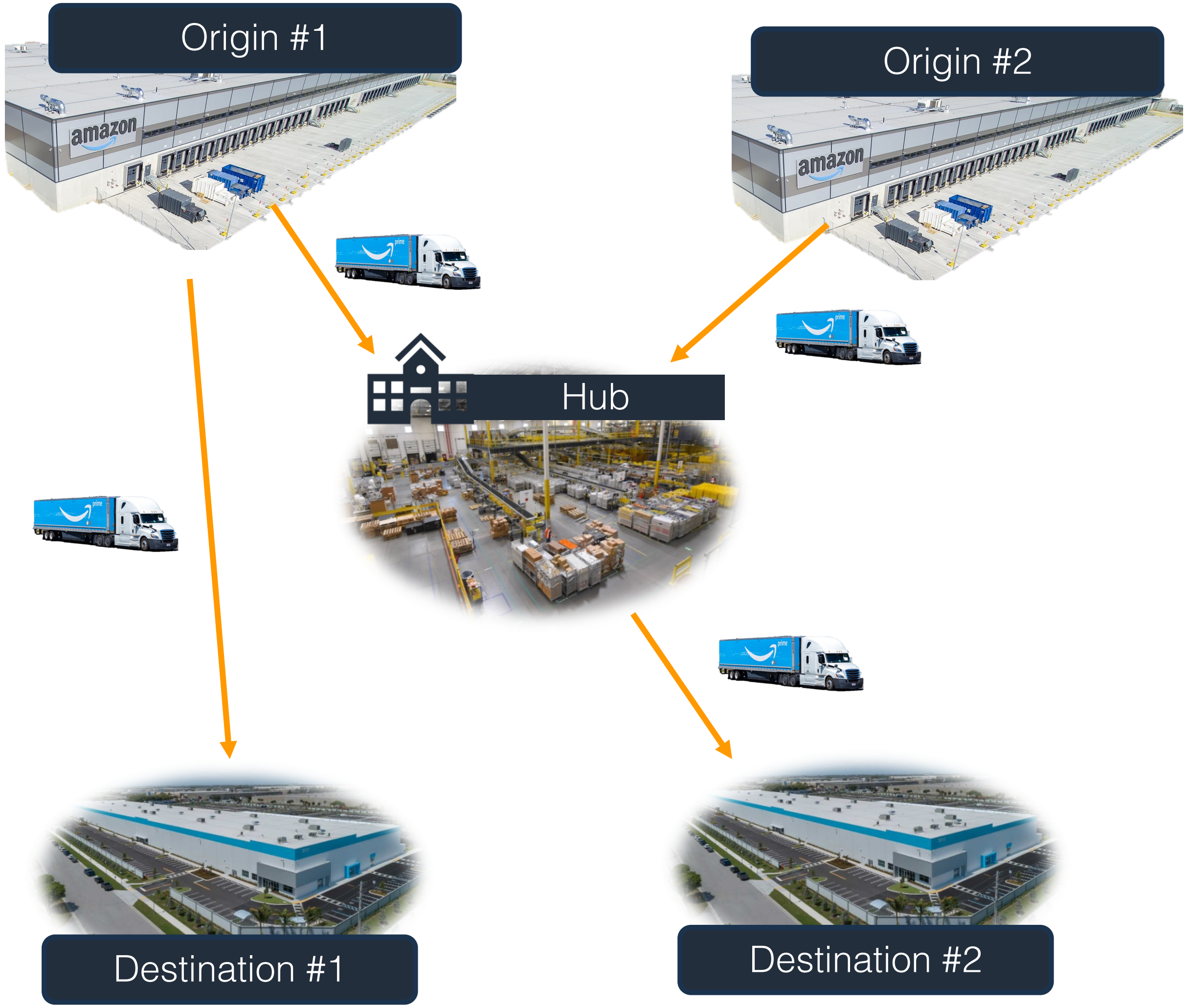
Network Design: Connectivity



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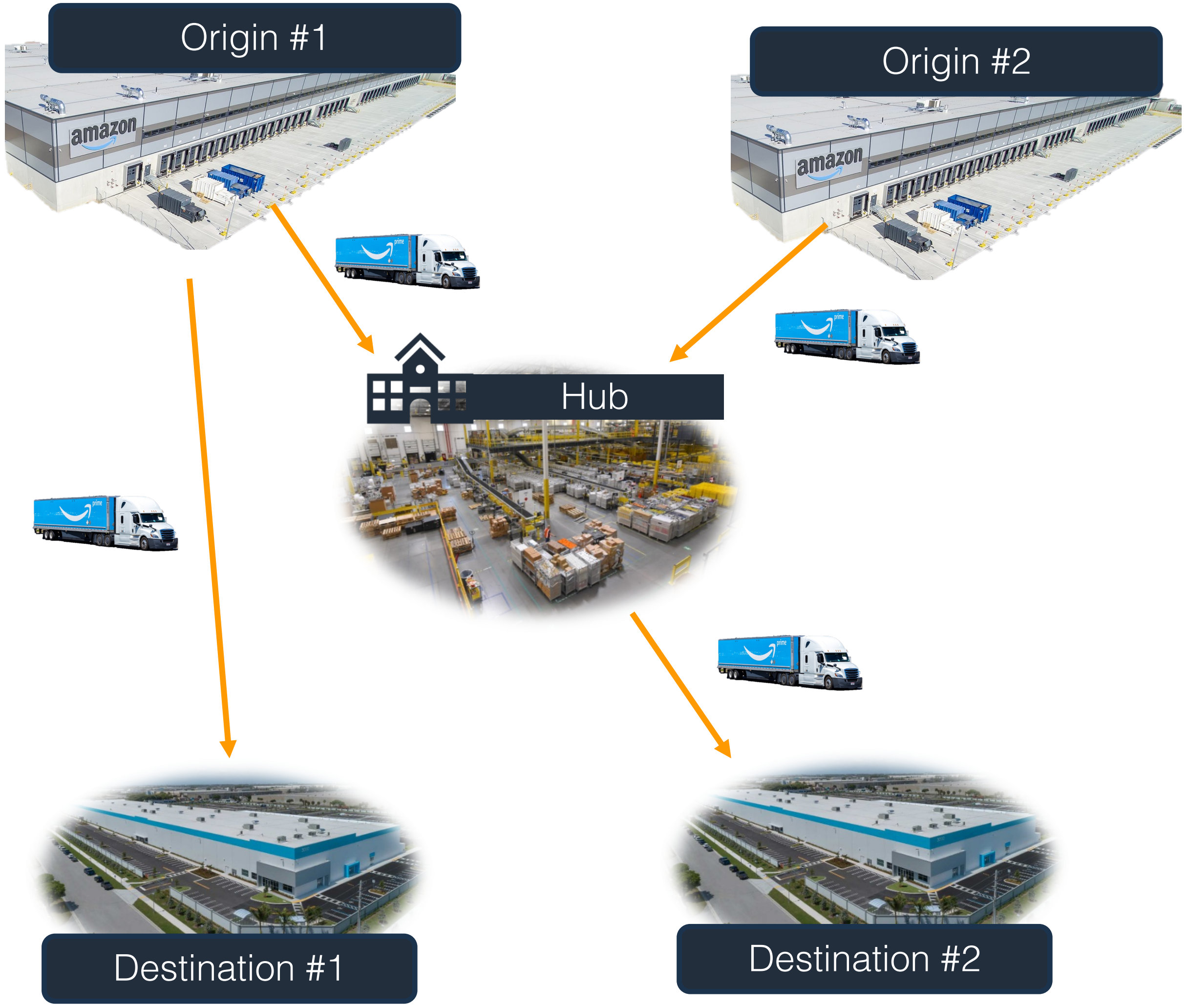
Network Design: Connectivity



$$\min_{\boldsymbol{p},\boldsymbol{v}} \quad \texttt{NetworkCost}(\boldsymbol{p},\boldsymbol{y})$$
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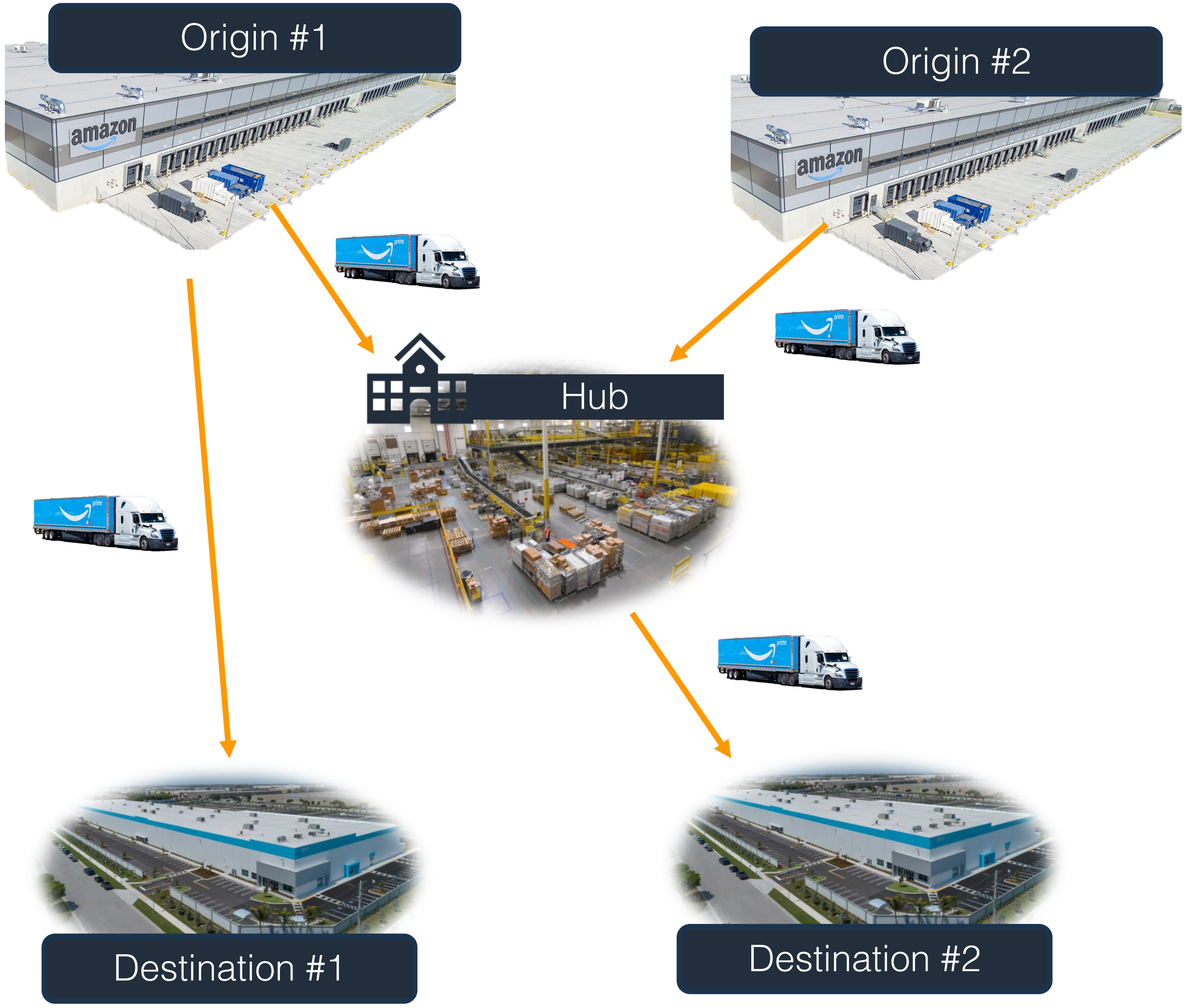
Network Design: Connectivity



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Path vector variable

Trucks vector variable

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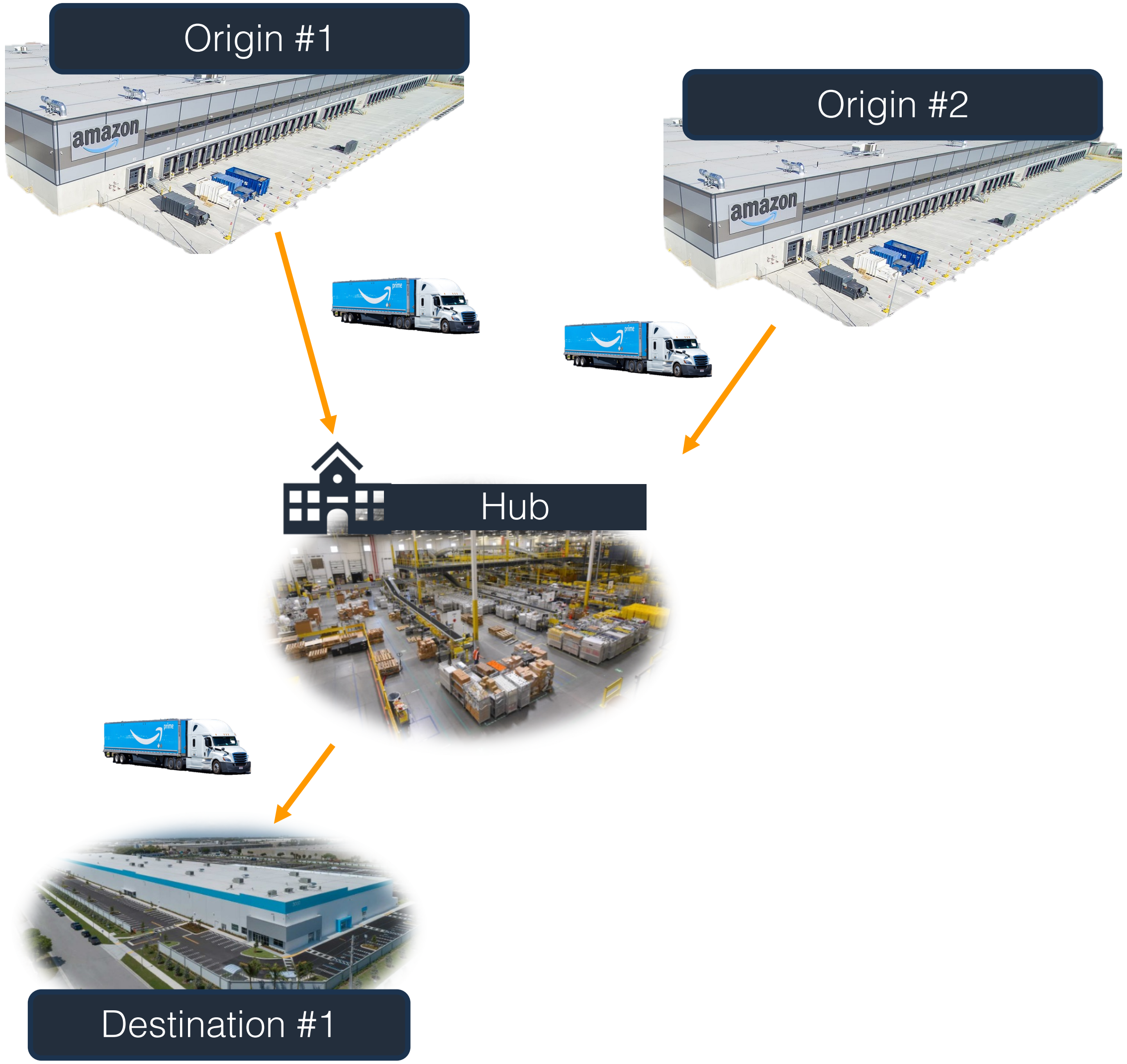
Network Design: Timing



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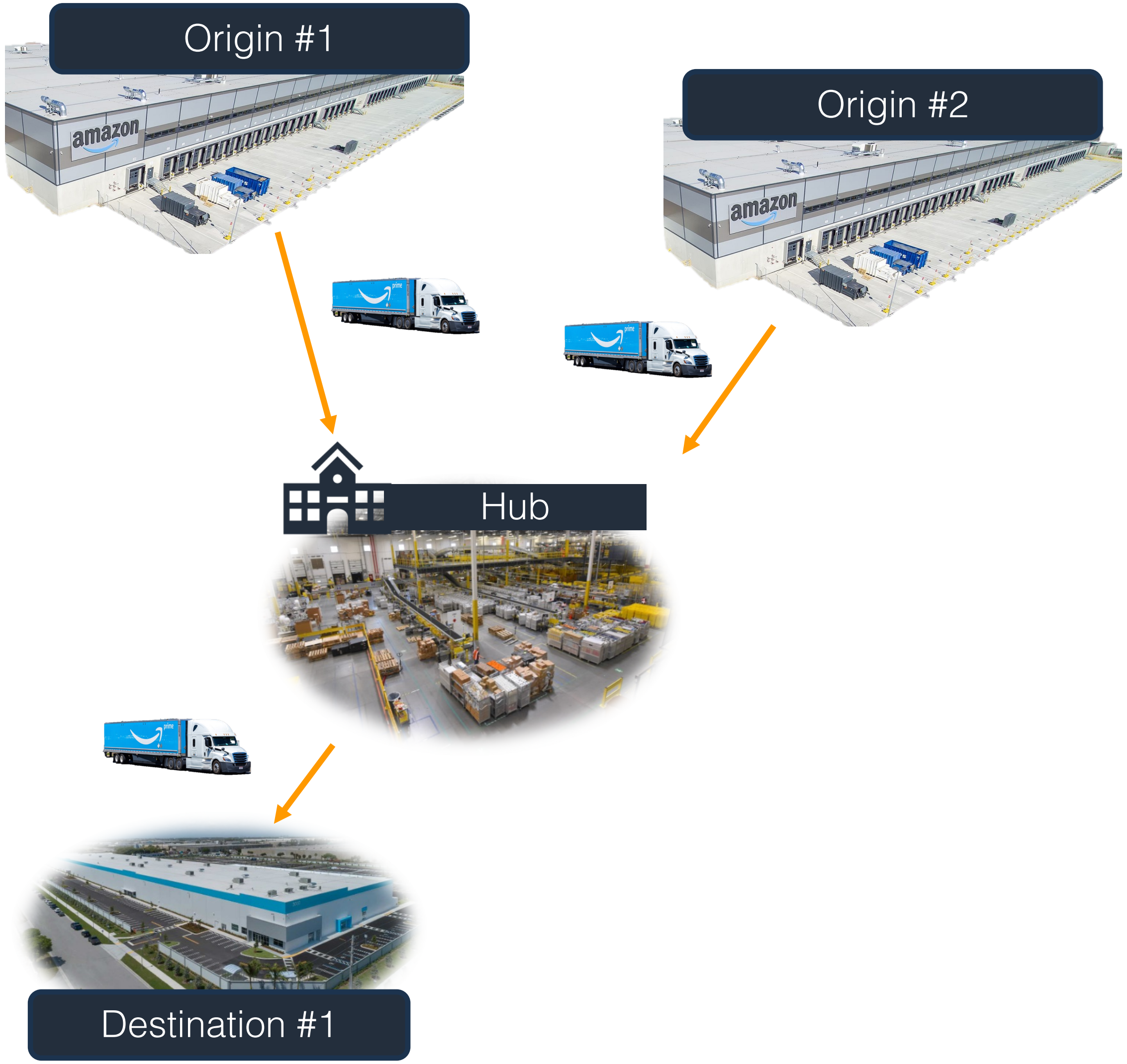

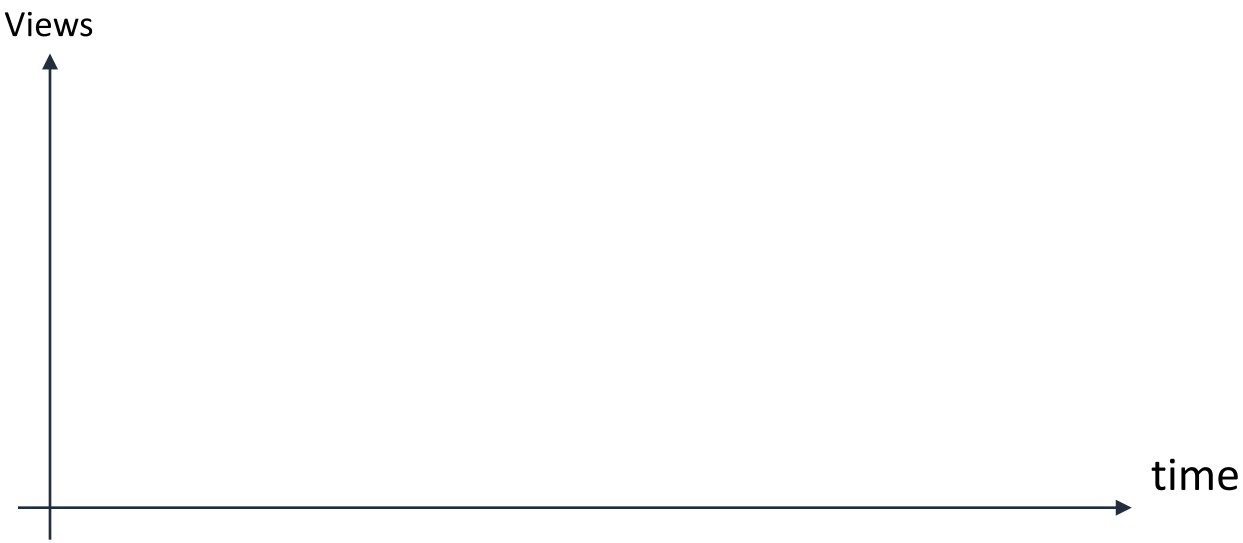

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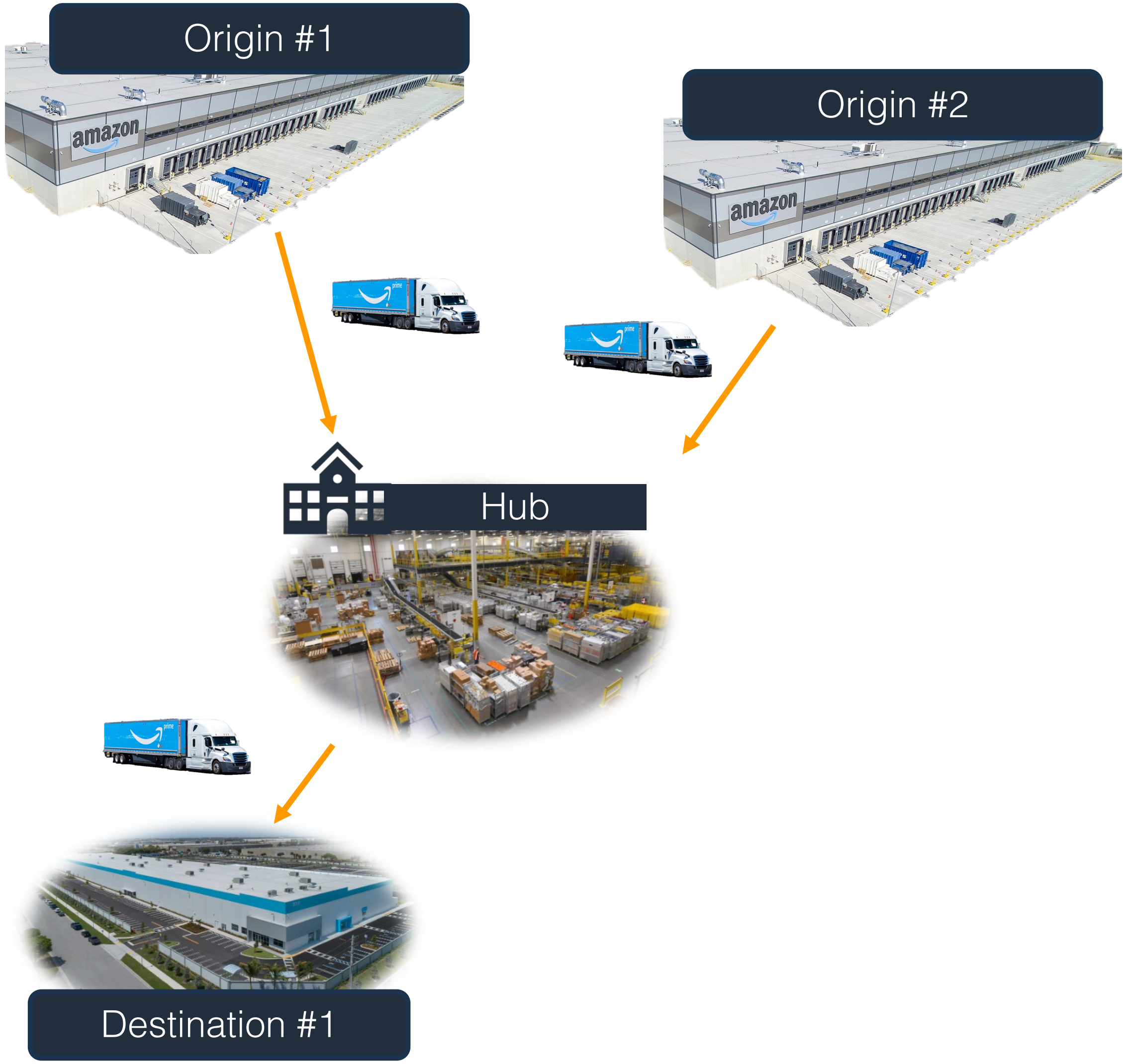

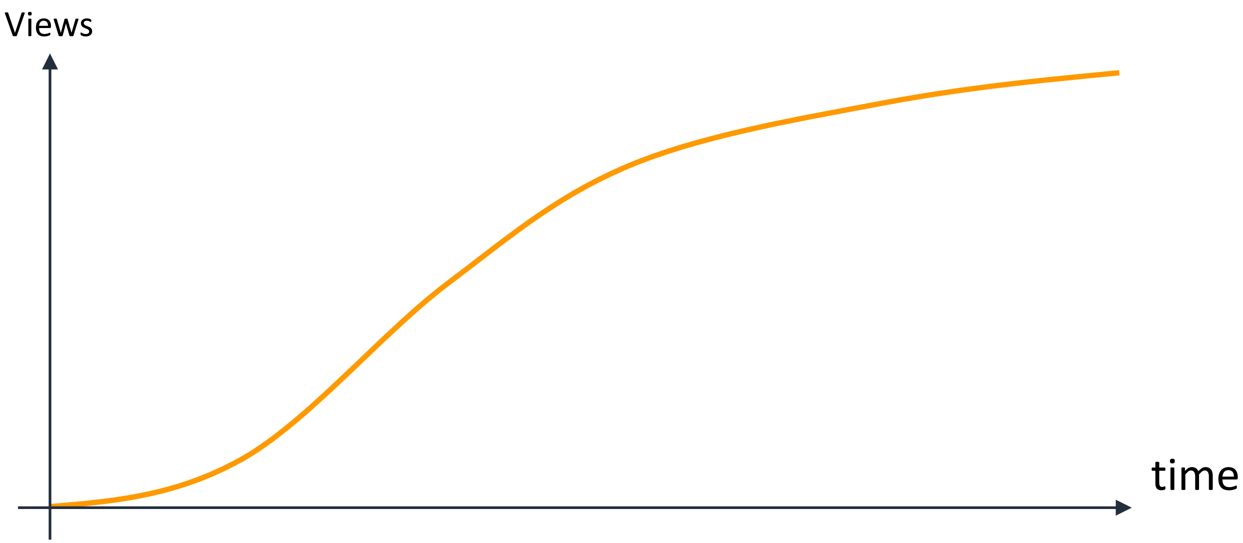

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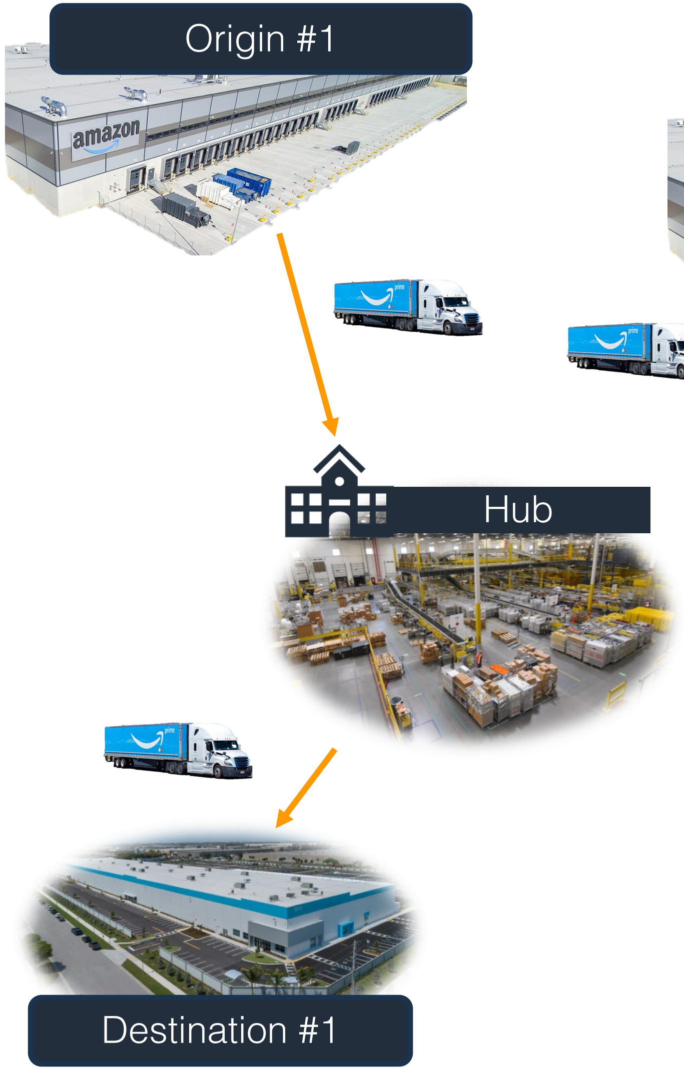

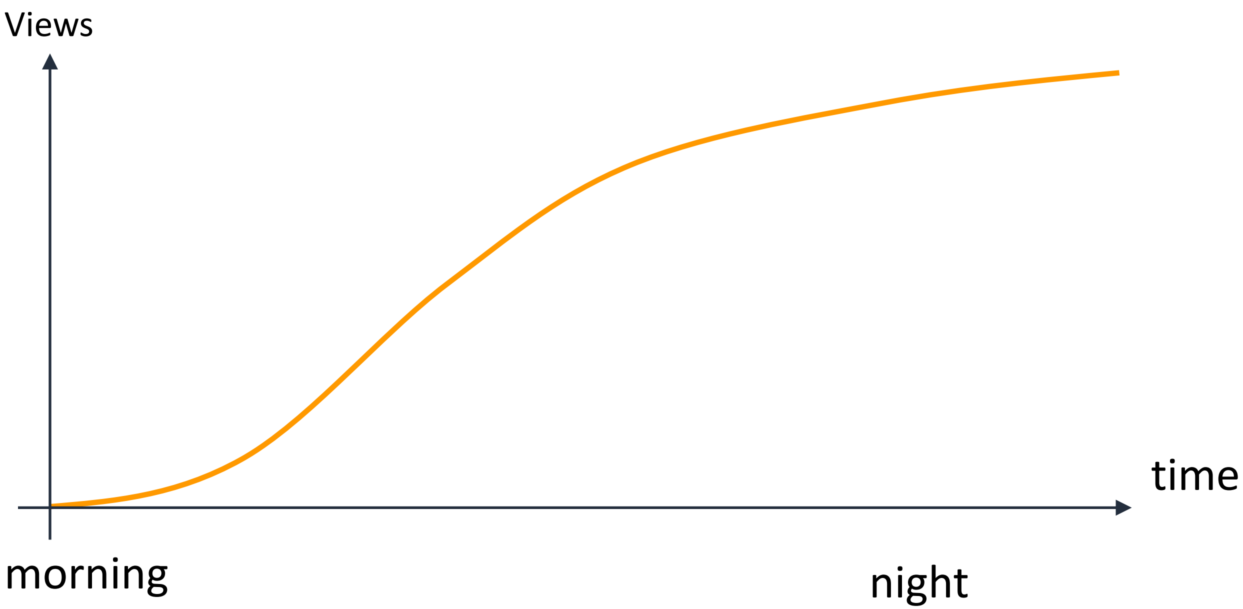

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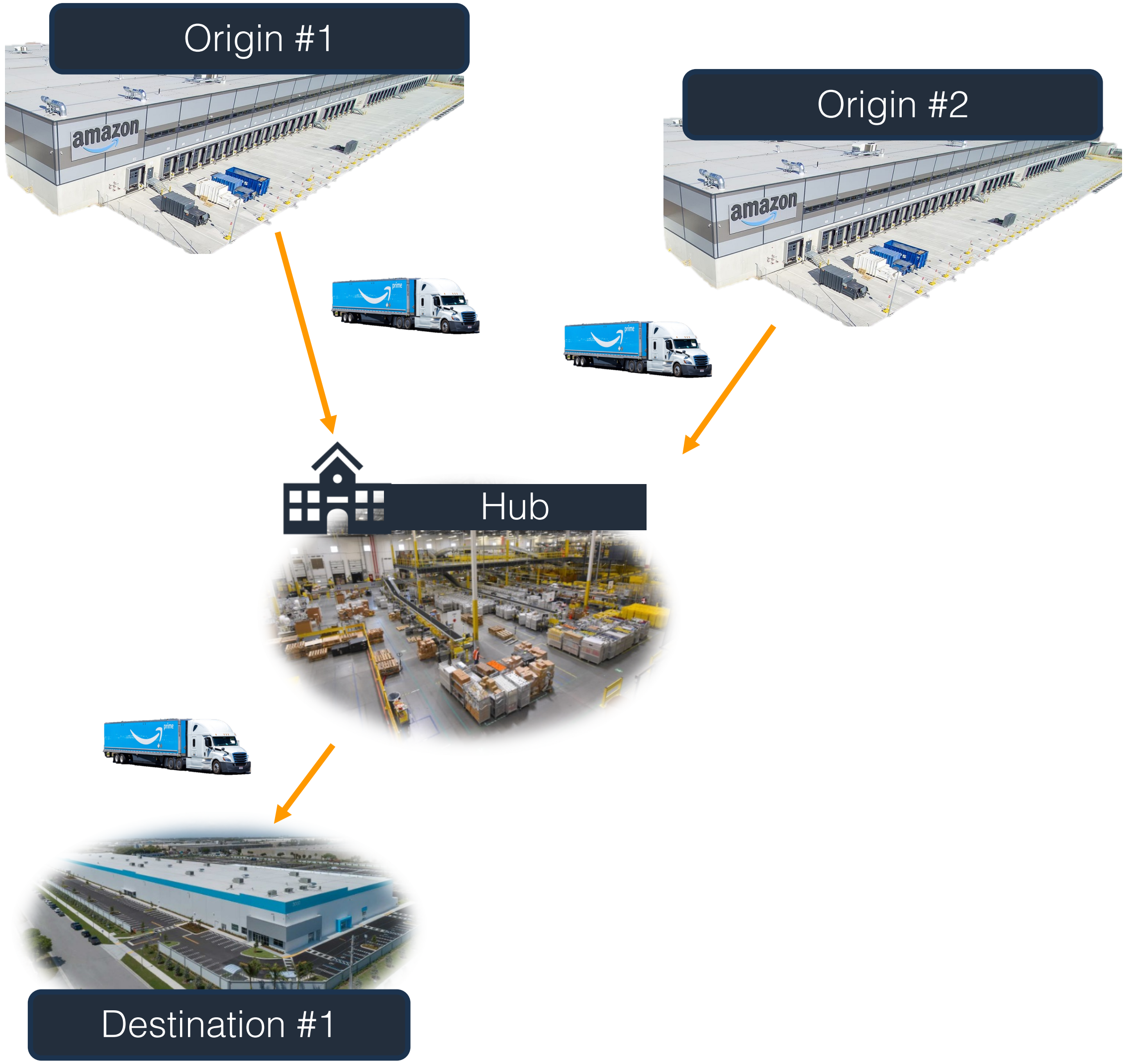

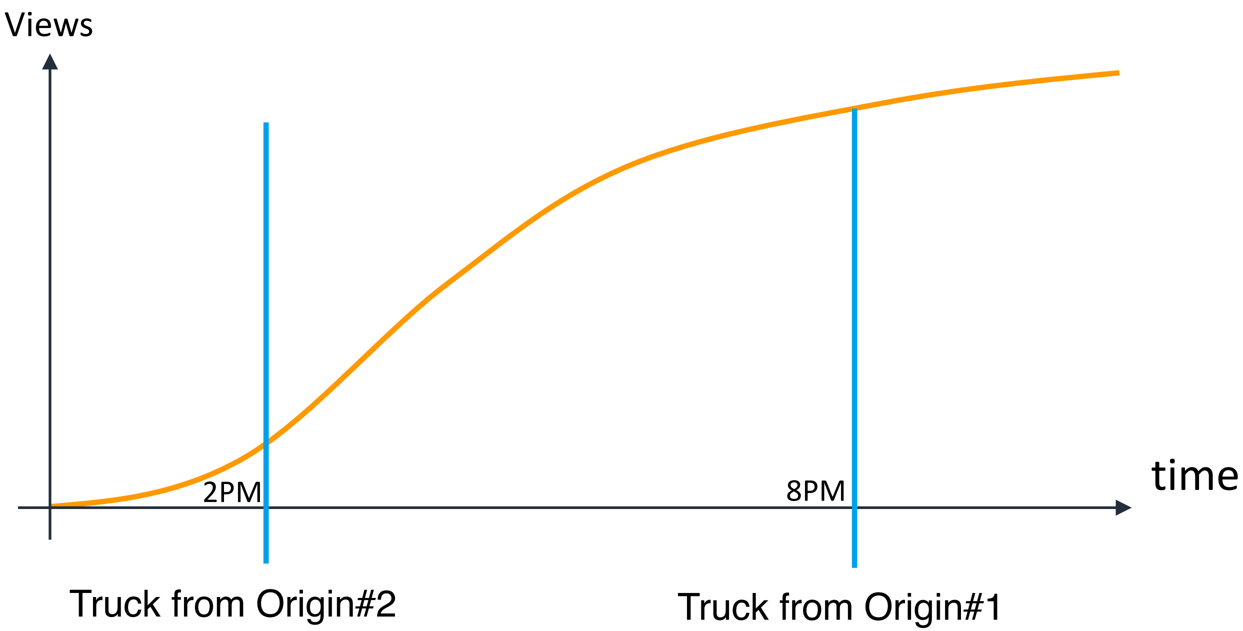

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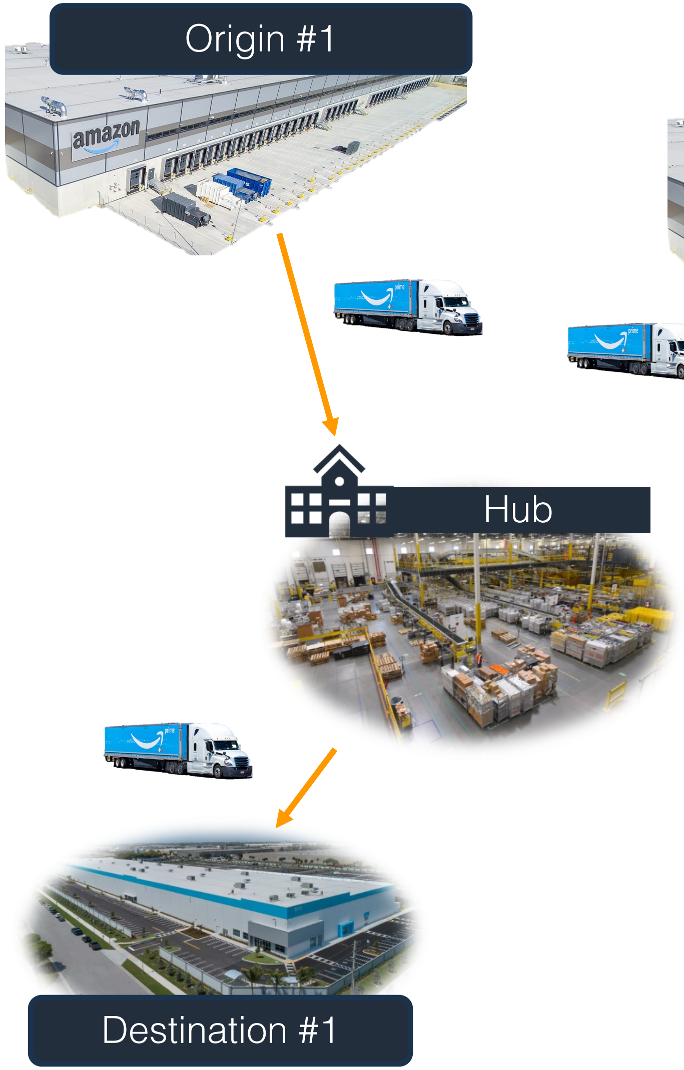

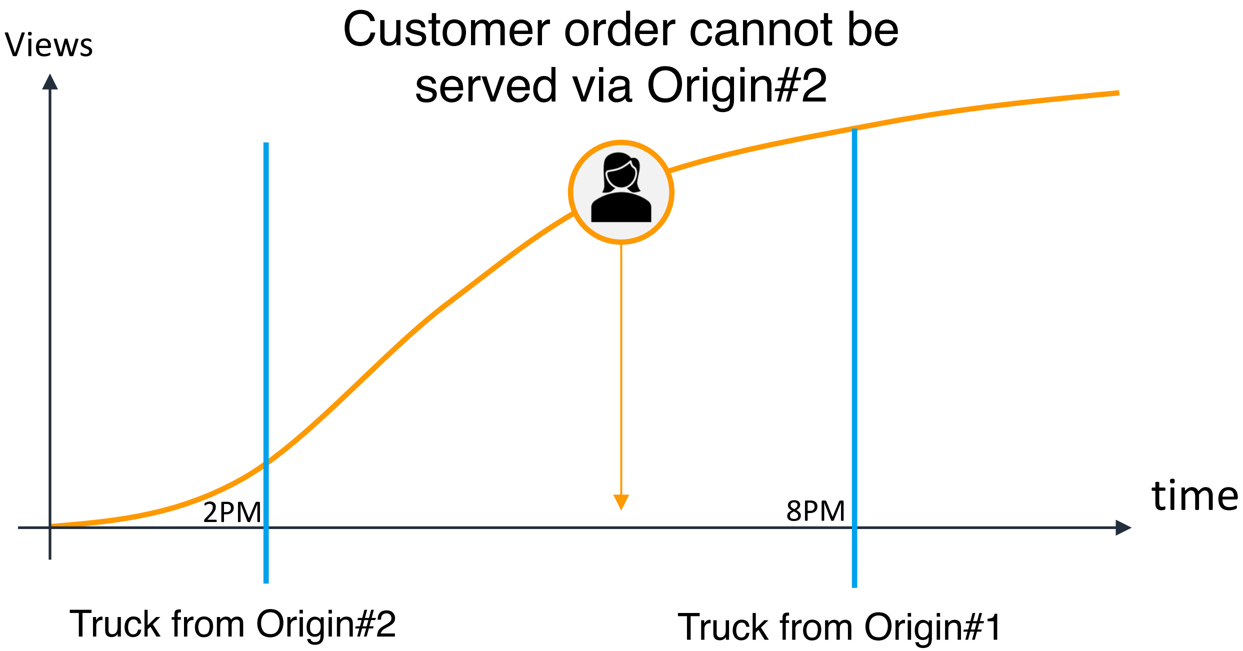

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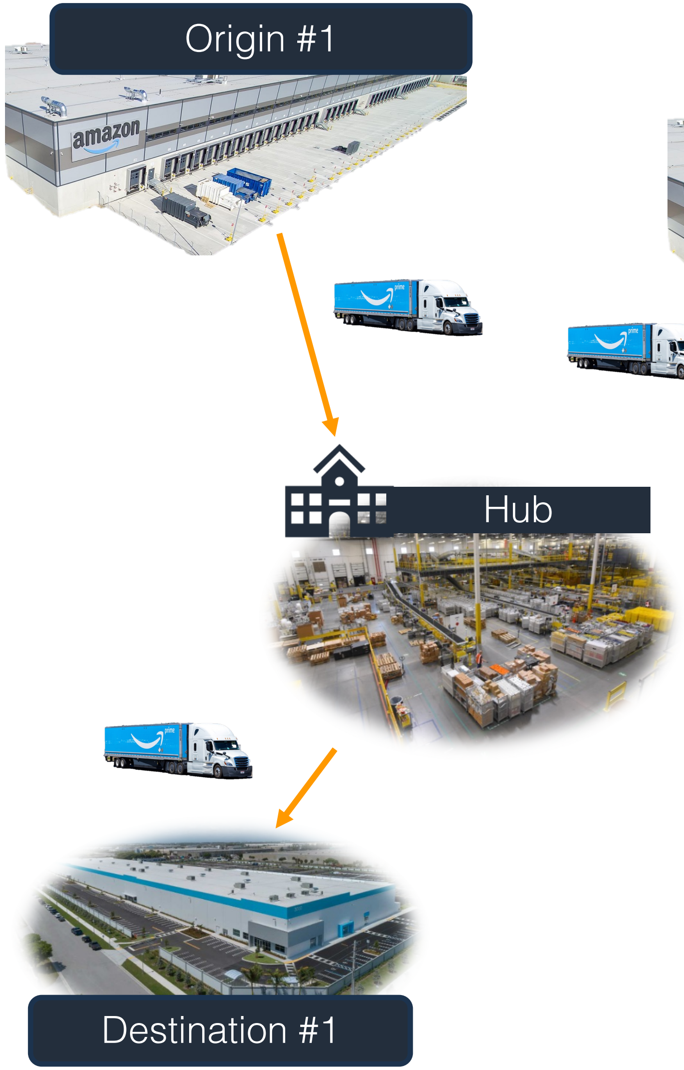

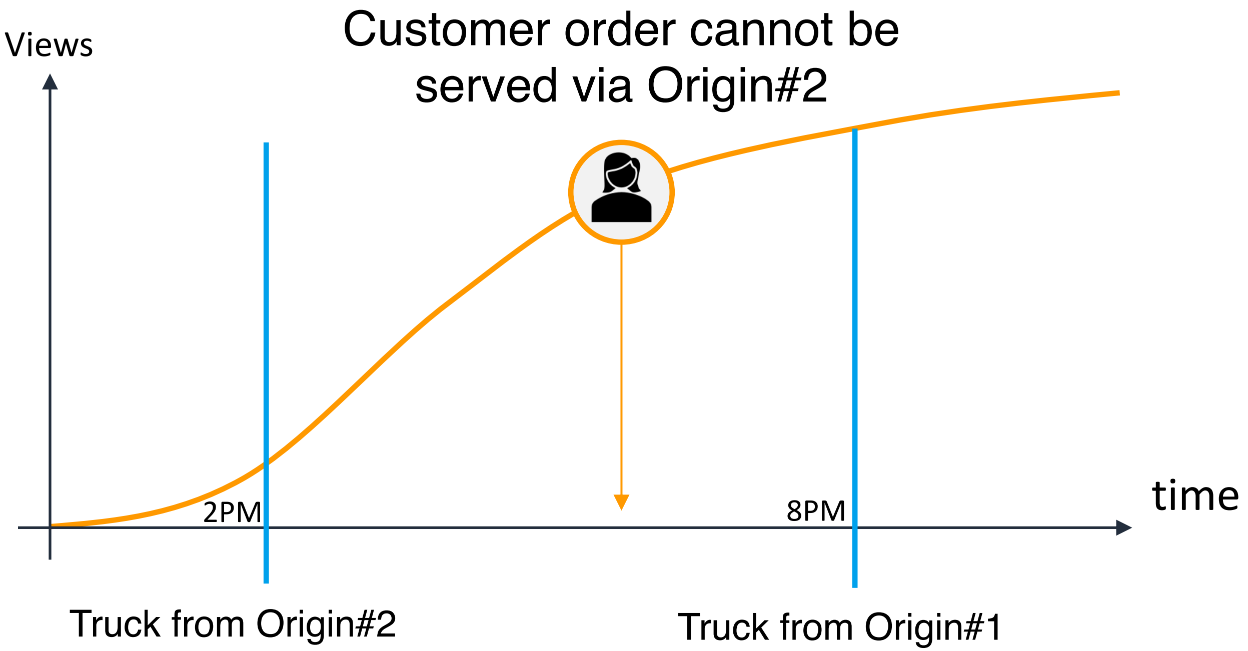

$$\max_{z} \quad \mathtt{Speed}(z)$$
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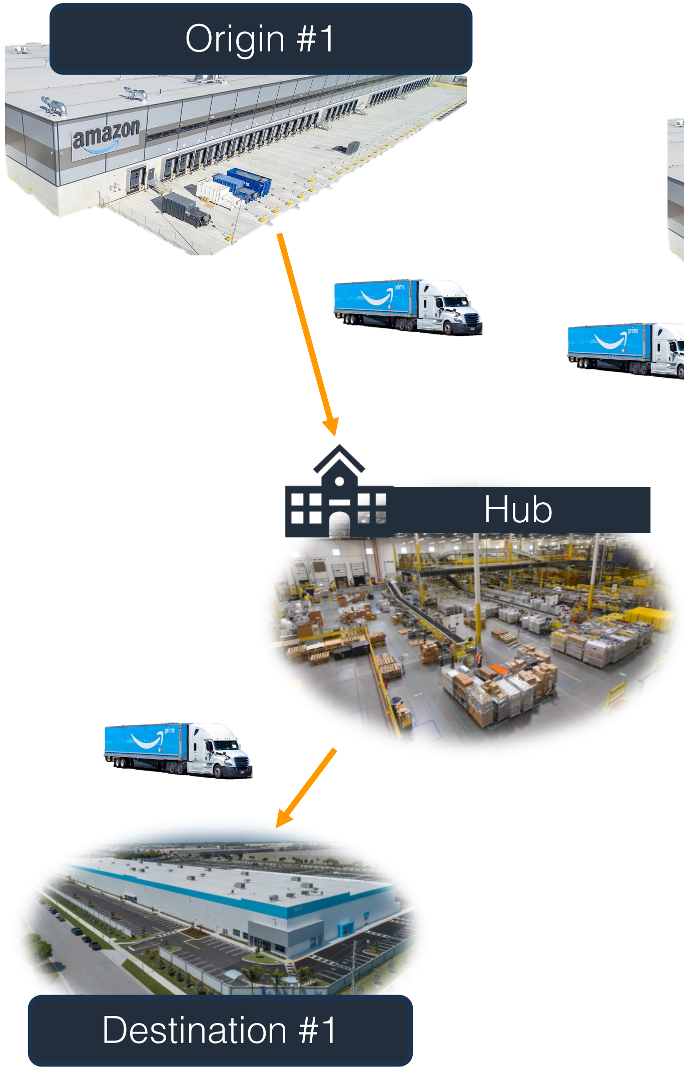

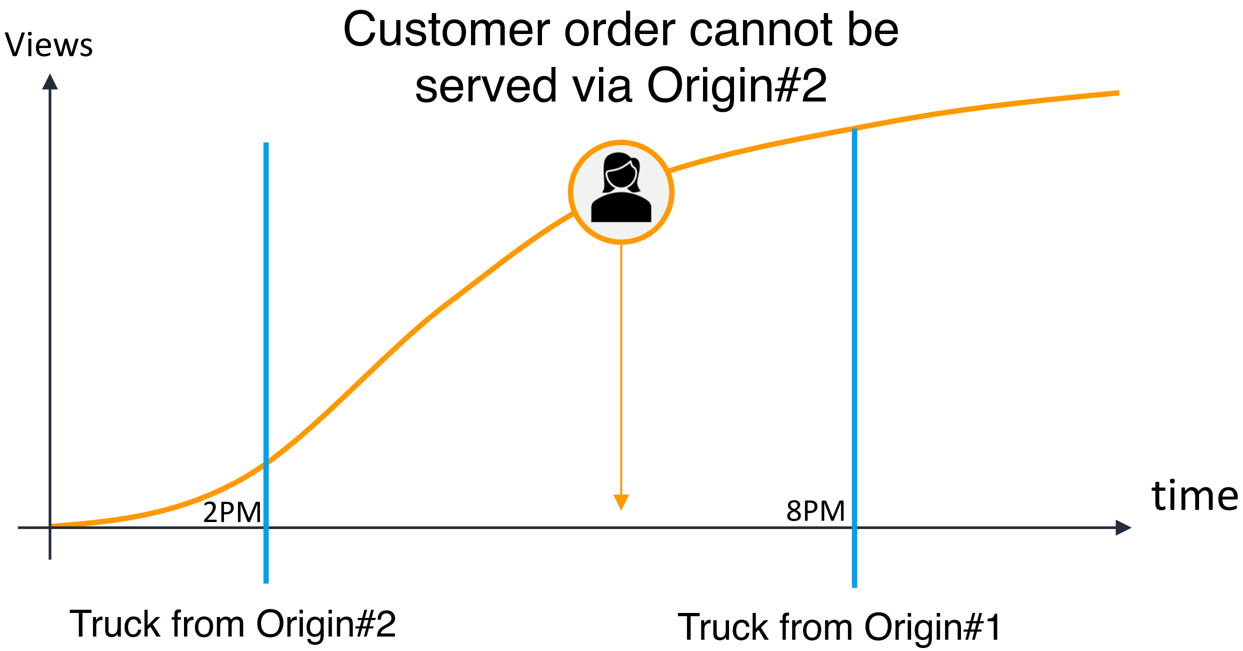

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Truck schedule vector variable

Dependency on connectivity

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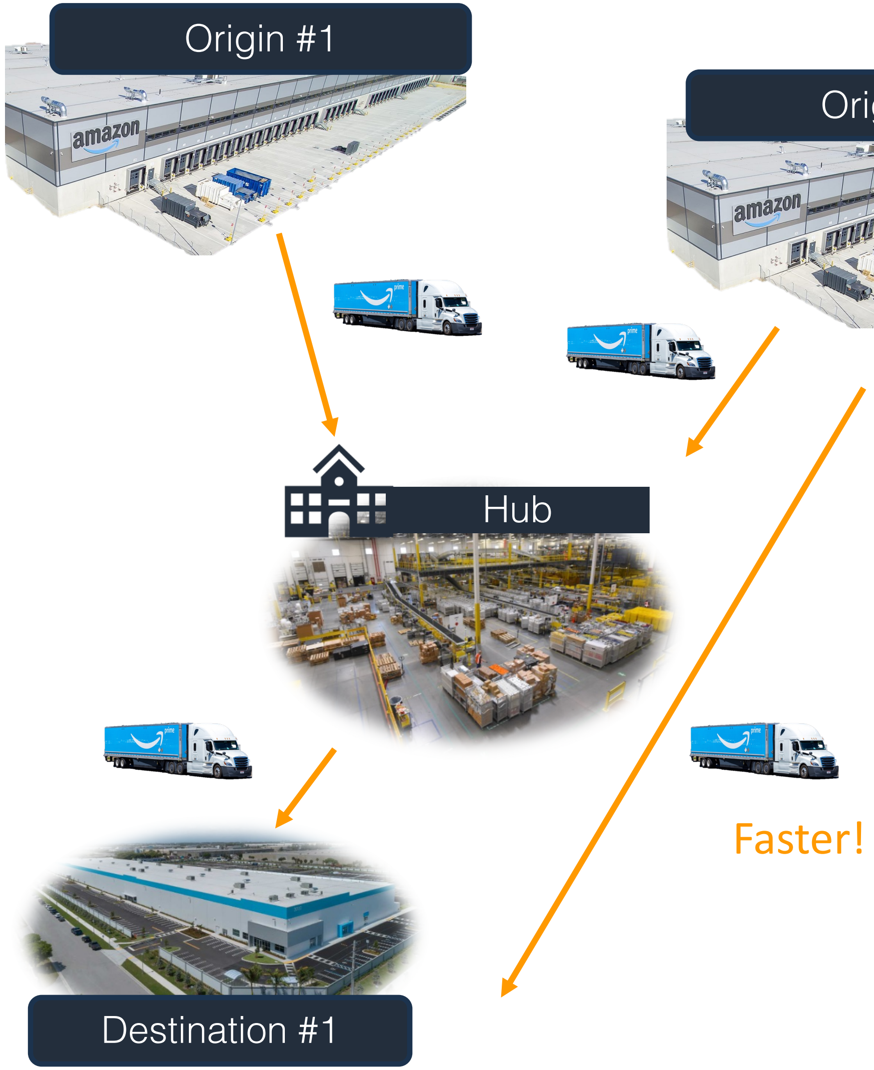

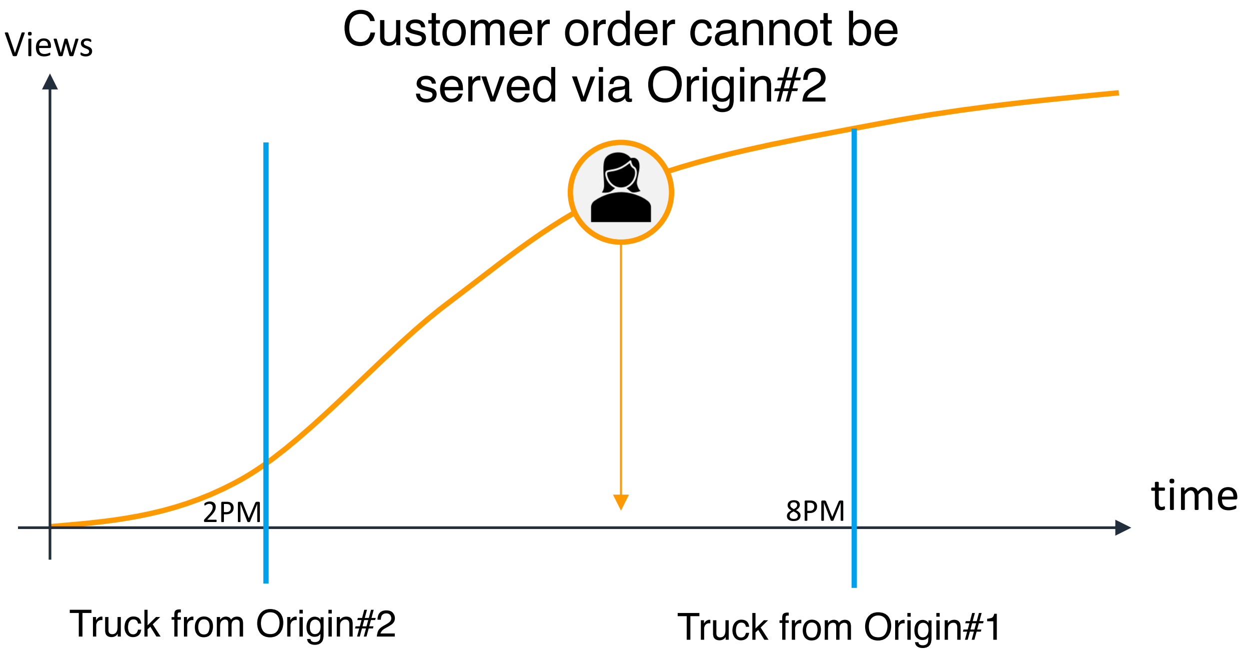

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Network Design: The joint problem

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{y},\boldsymbol{z}} \quad & \texttt{NetworkCost}(\boldsymbol{p},\boldsymbol{y}) - \texttt{Speed}(\boldsymbol{z}) \\ \text{s.t.} \quad & (\boldsymbol{p},\boldsymbol{y}) \in \texttt{FeasibleNetwork} \\ & \boldsymbol{z} \in \texttt{FeasibleSchedule}(\boldsymbol{p}) \end{aligned}$$

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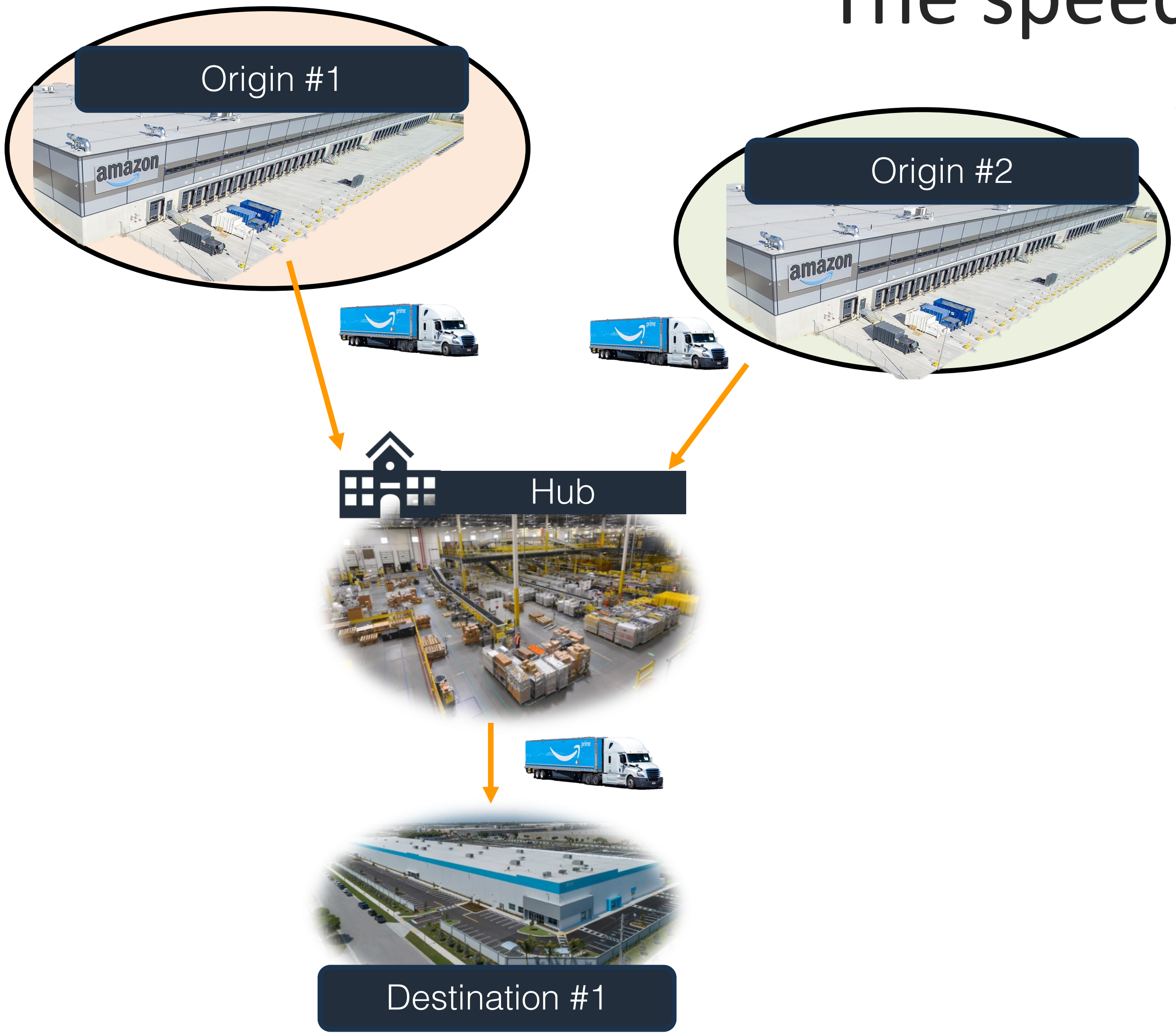

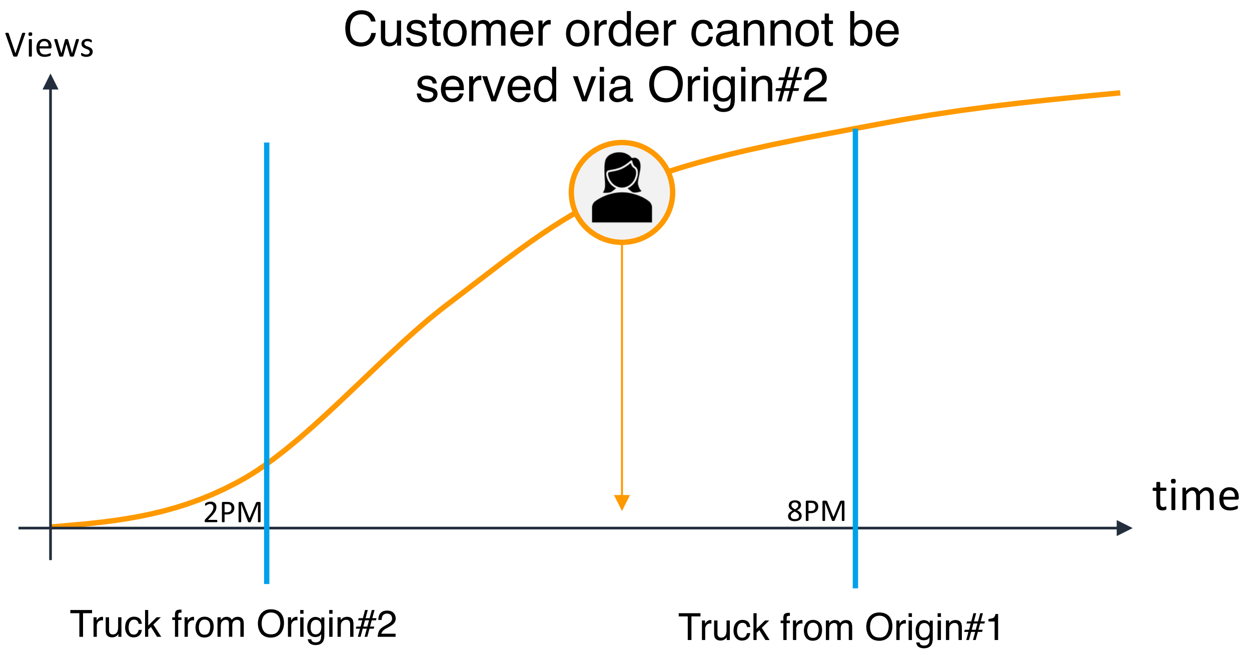

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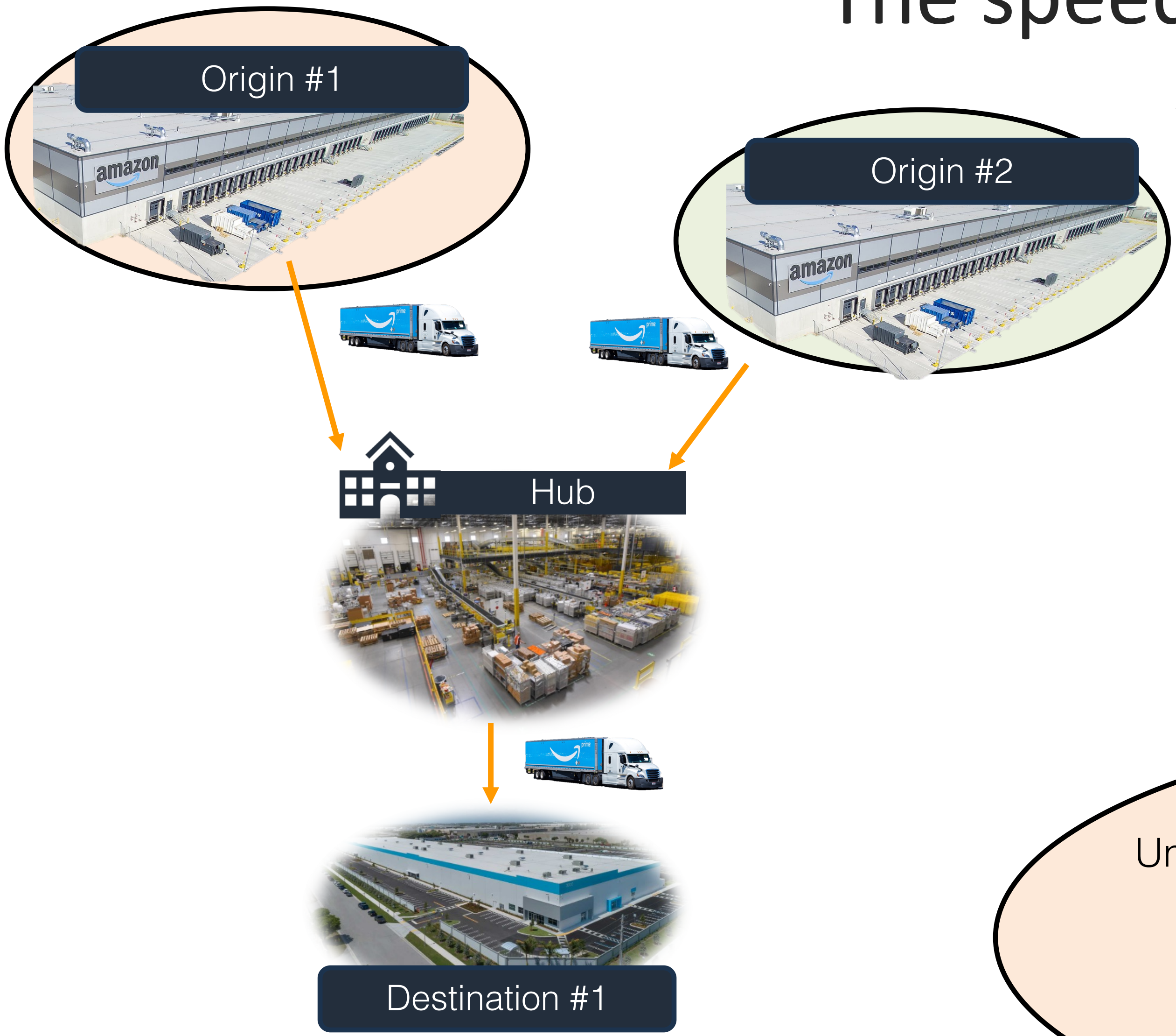

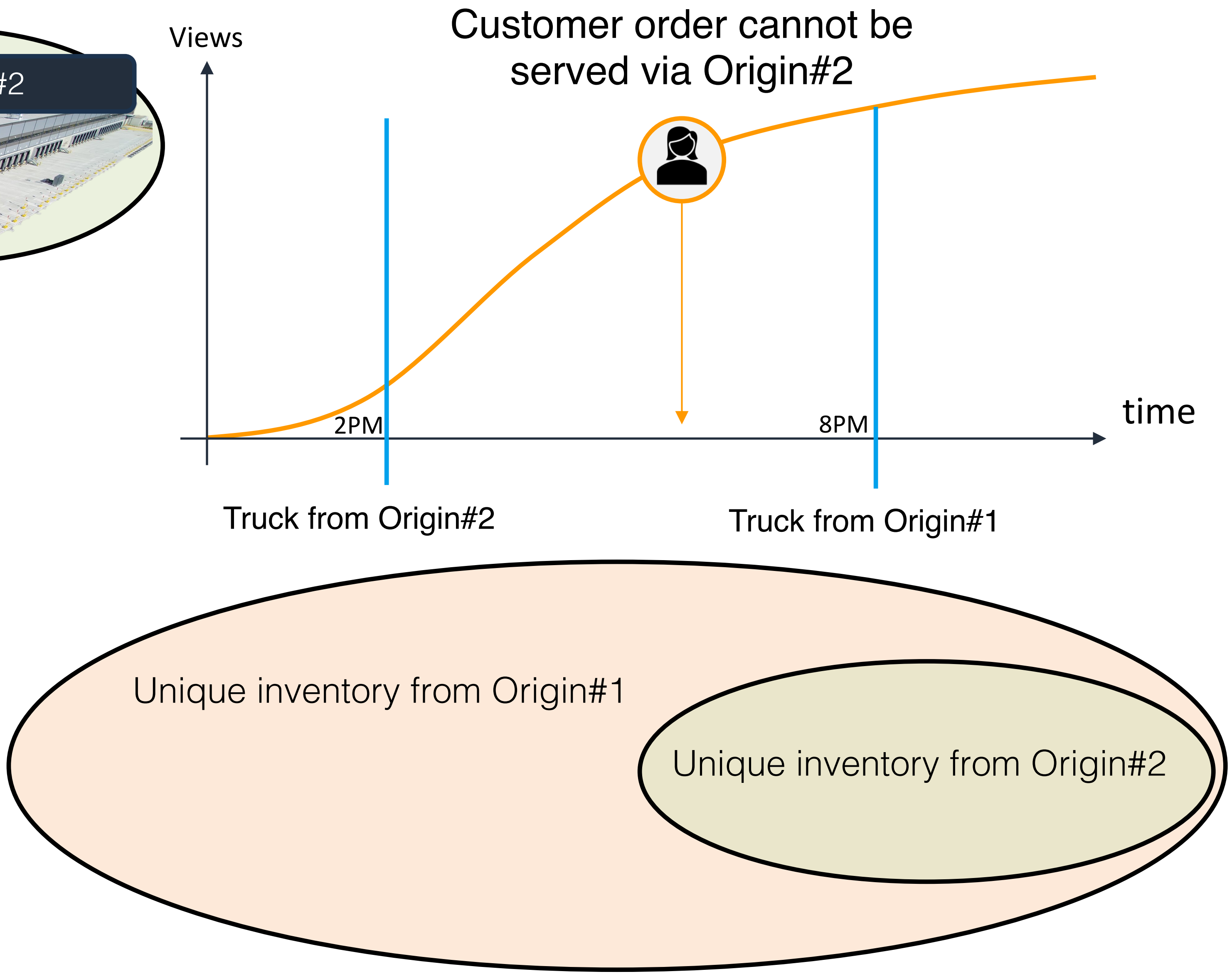

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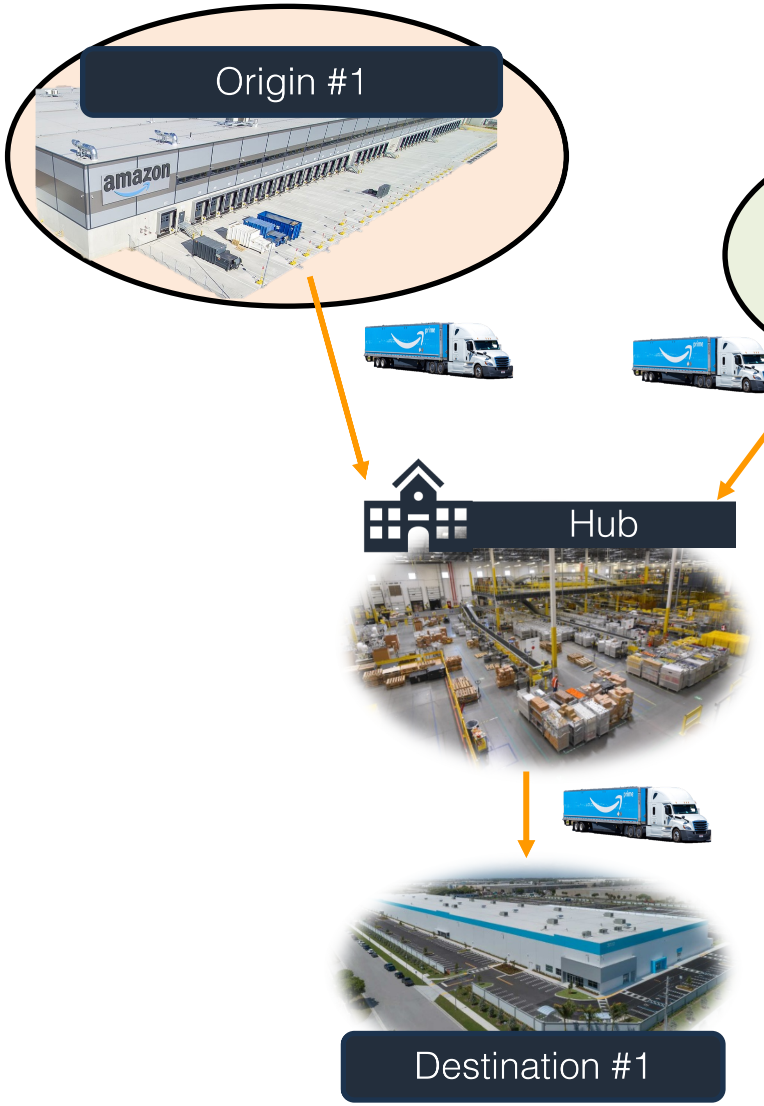

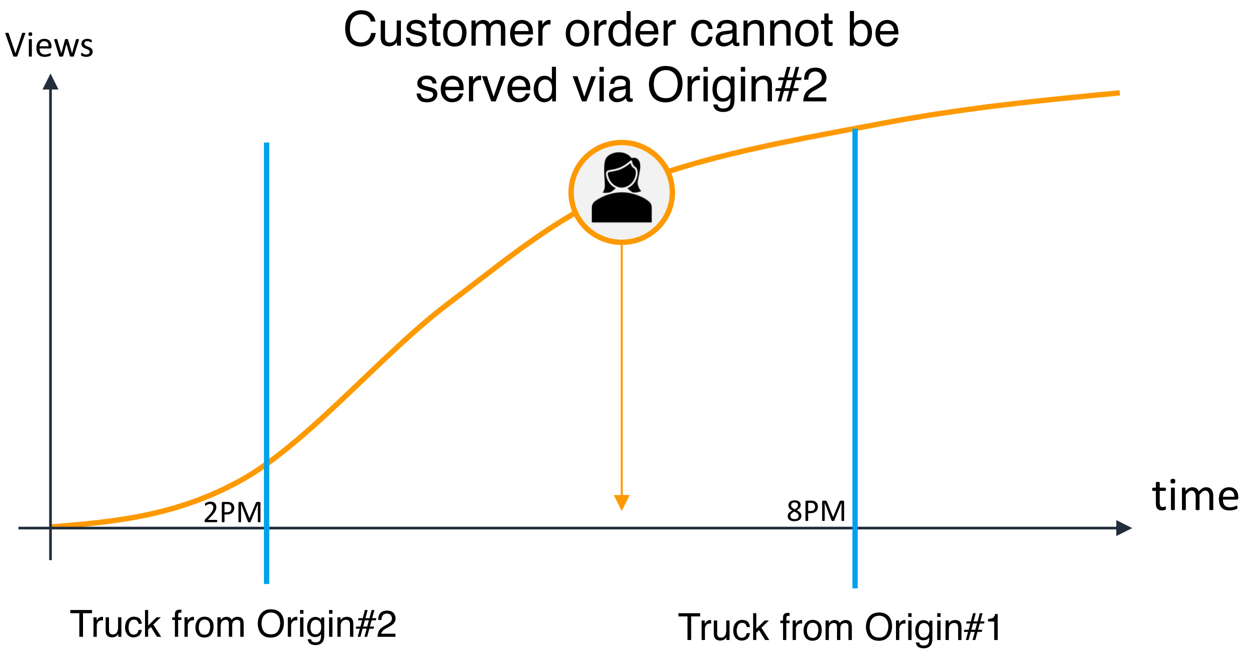

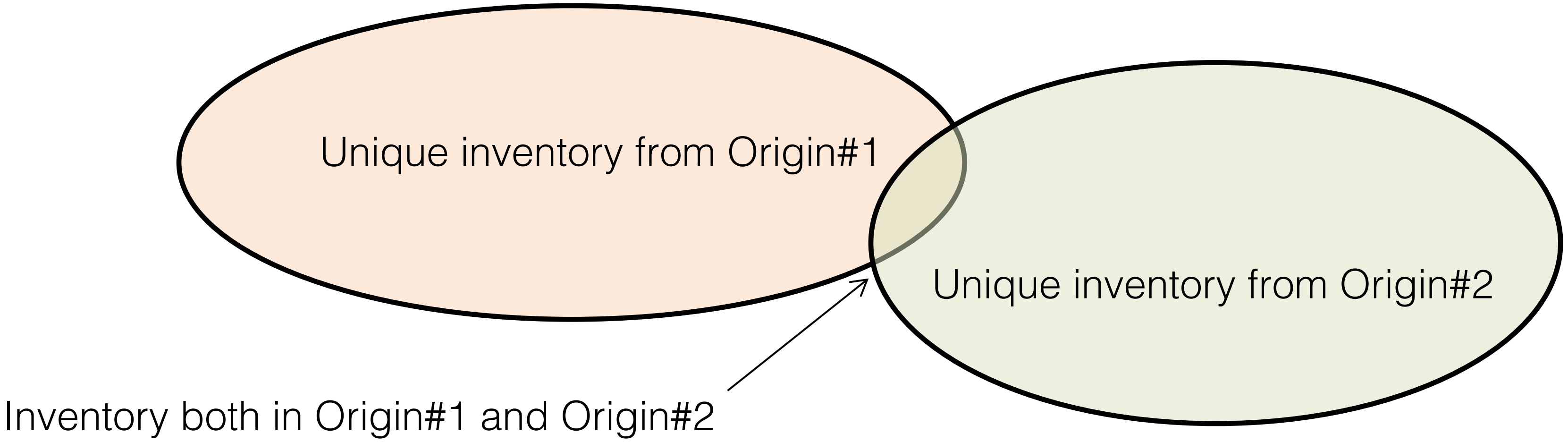

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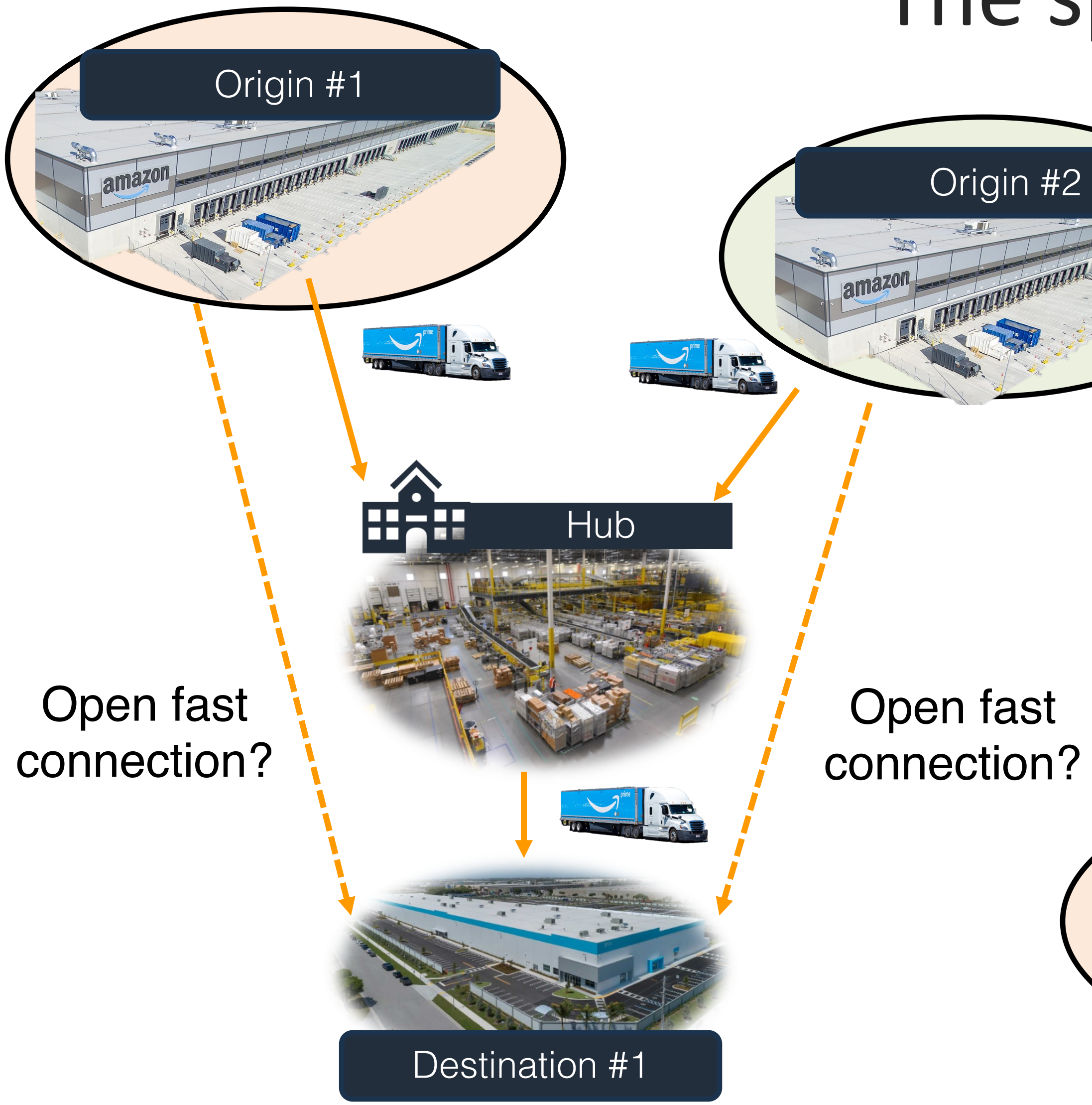

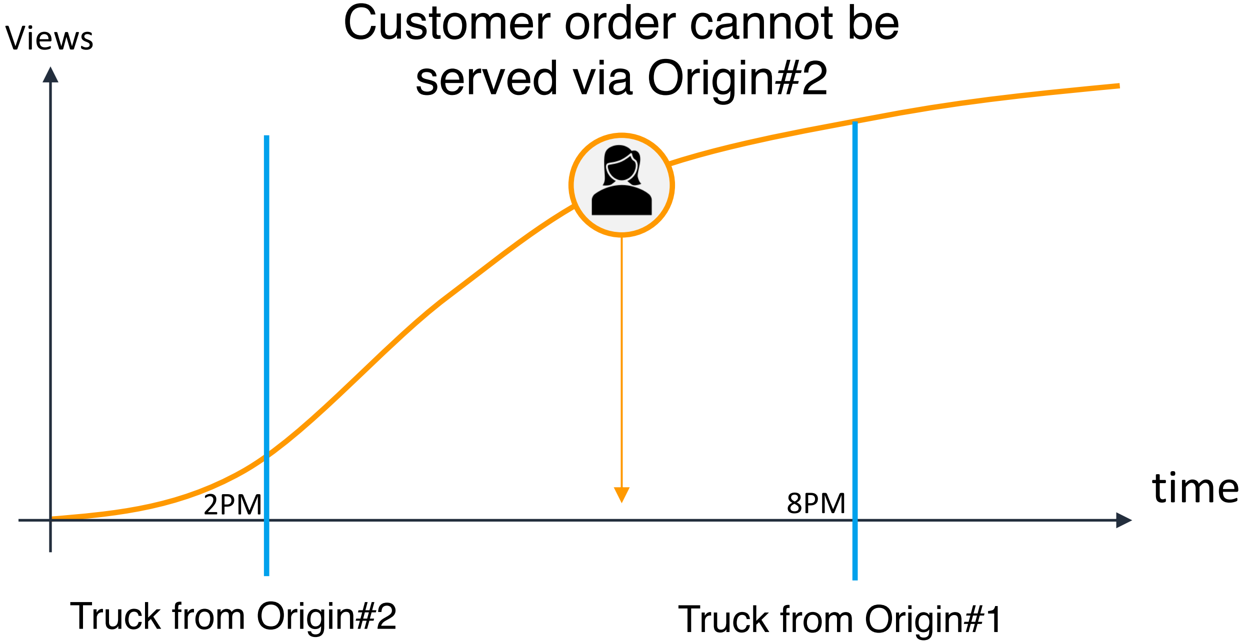

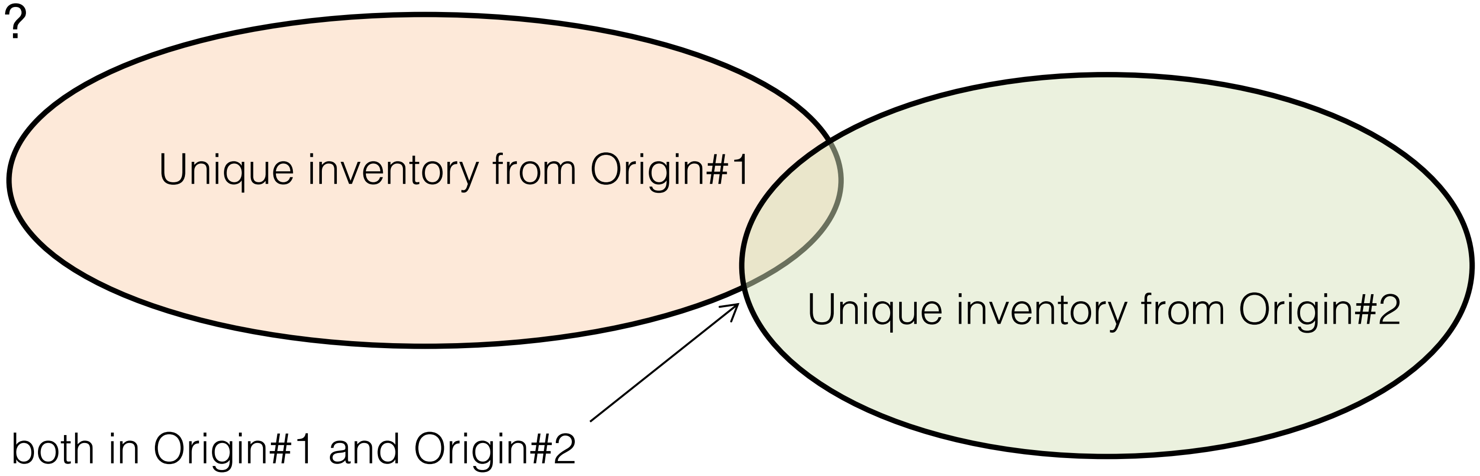

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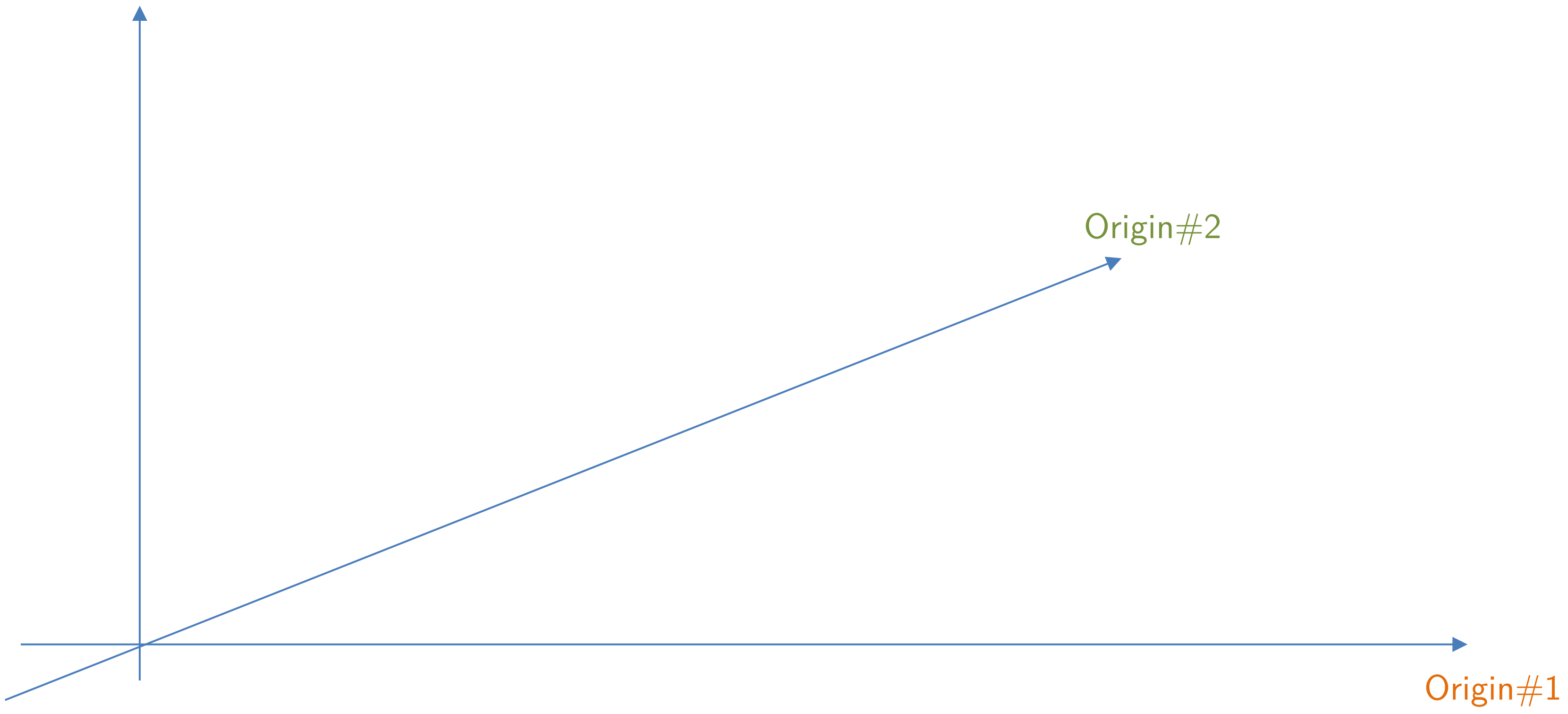
The speed objective

$\texttt{Speed}(z)$ = unique items that can be delivered in 1-day



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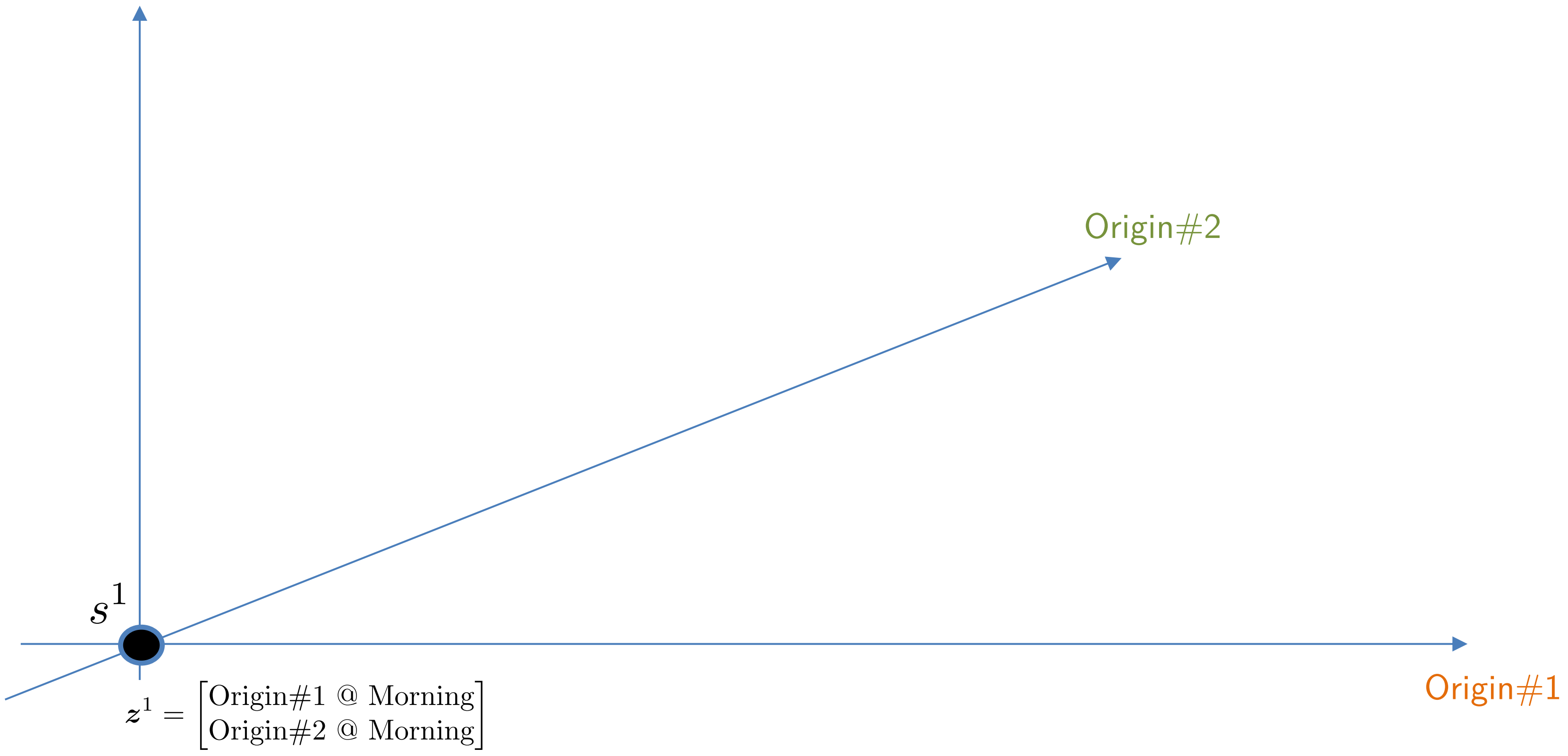

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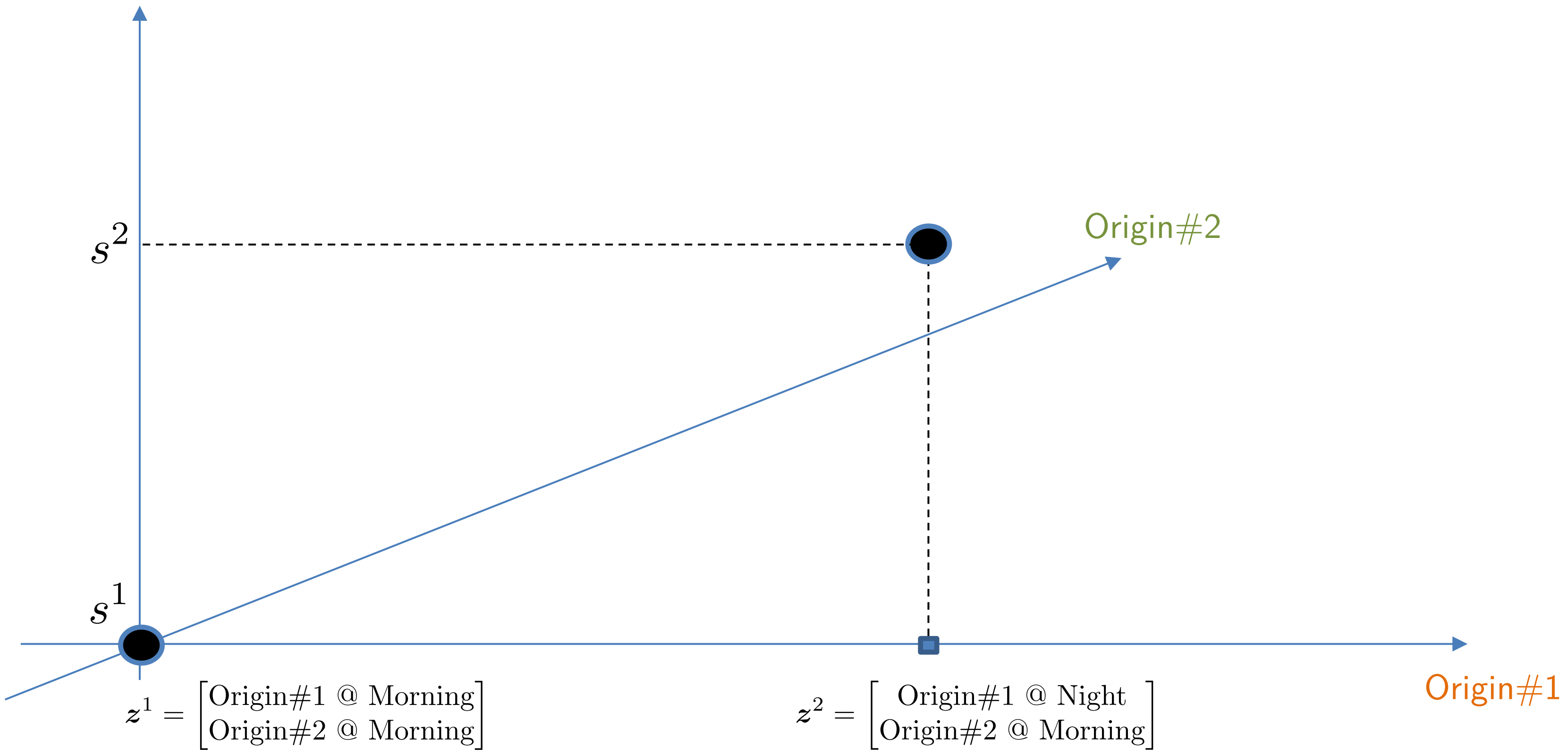

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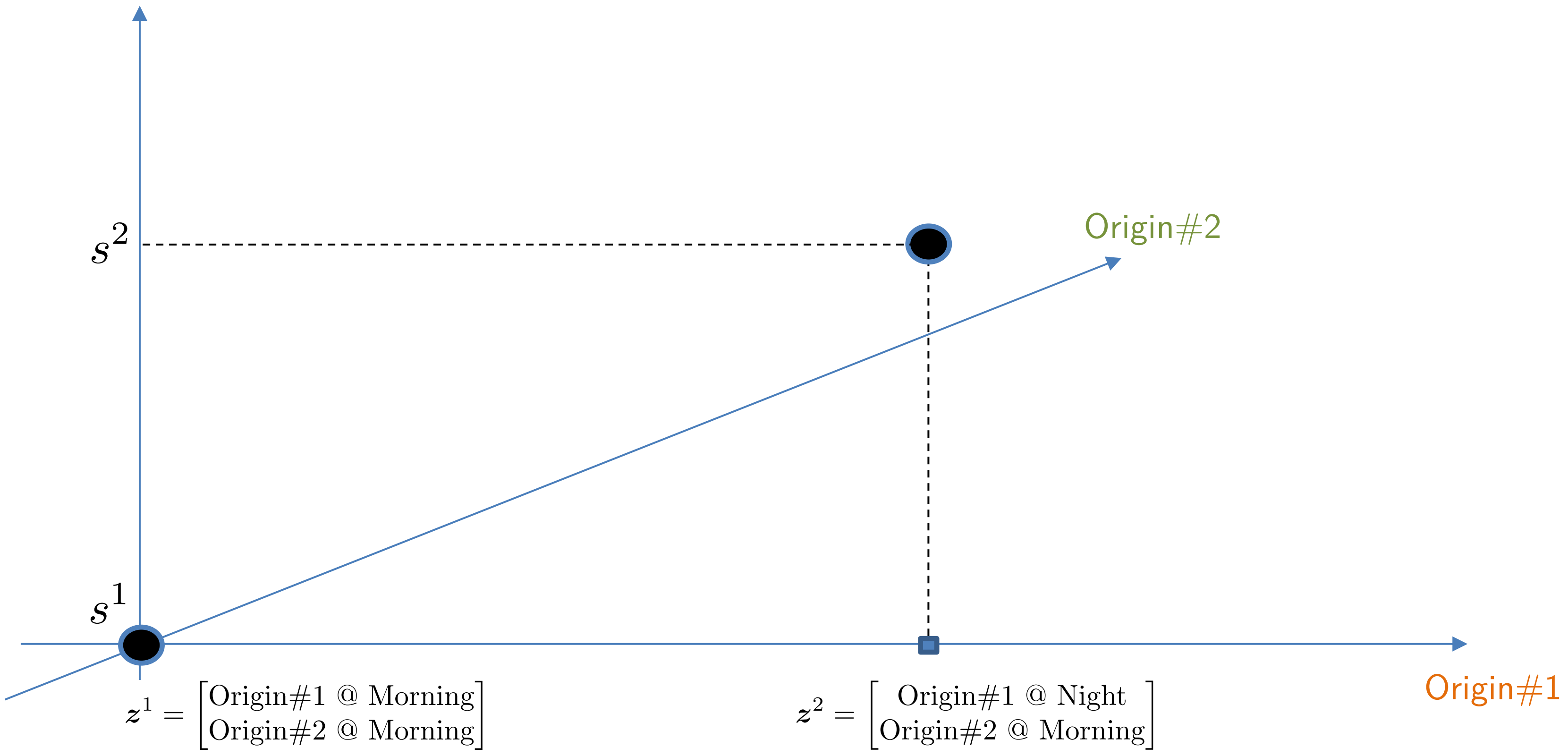

$\boxed{\texttt{Speed}(z) \text{ can be computed via a look-up table}}$

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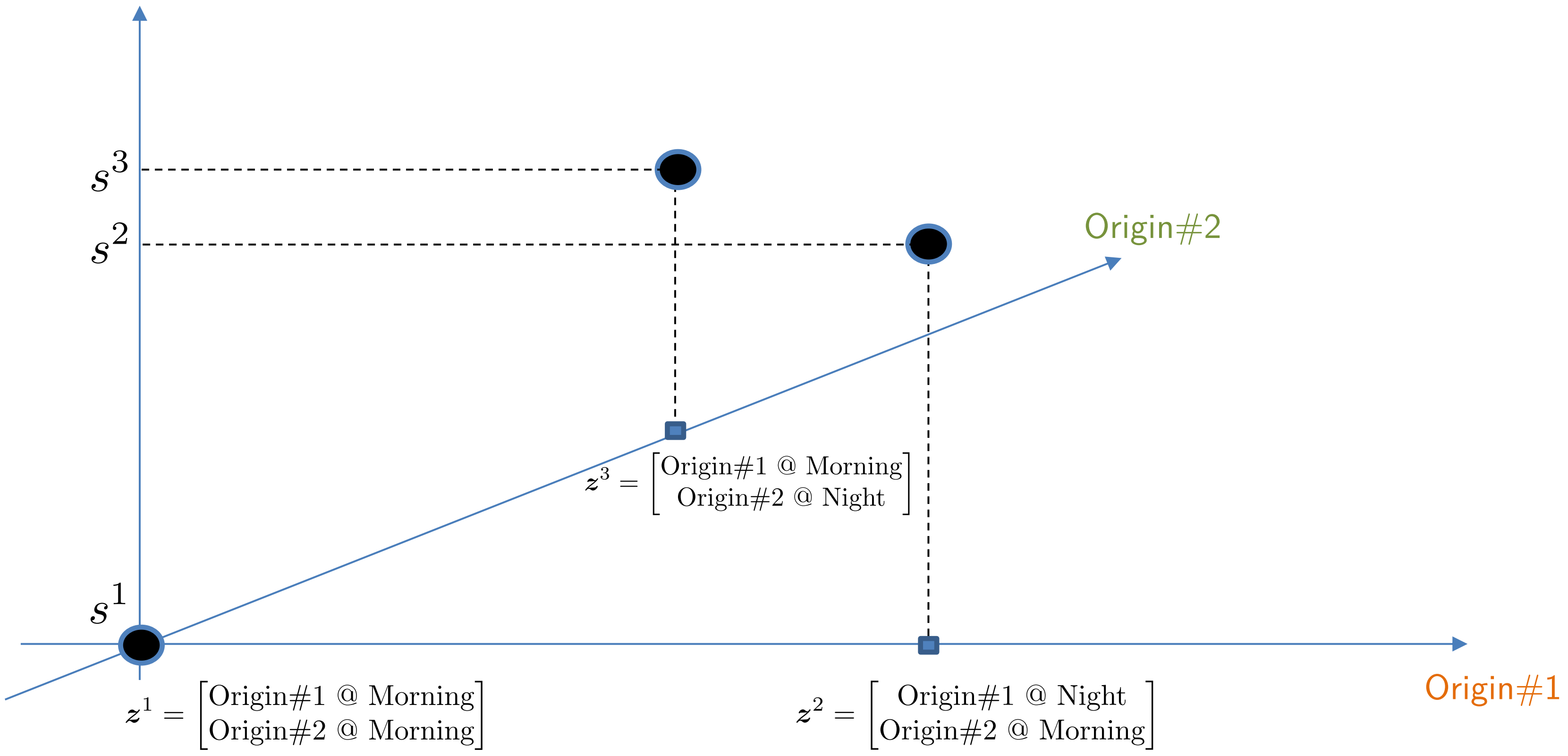

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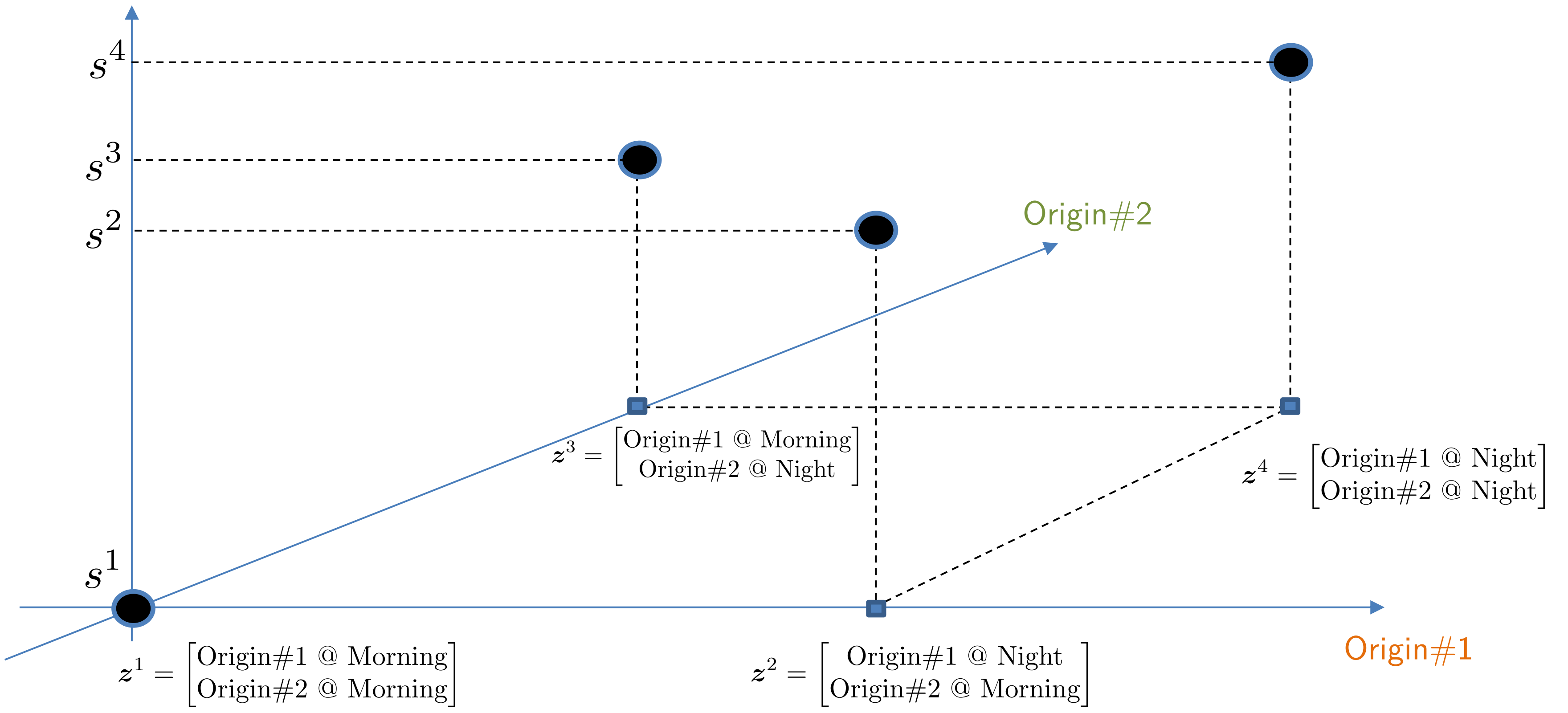

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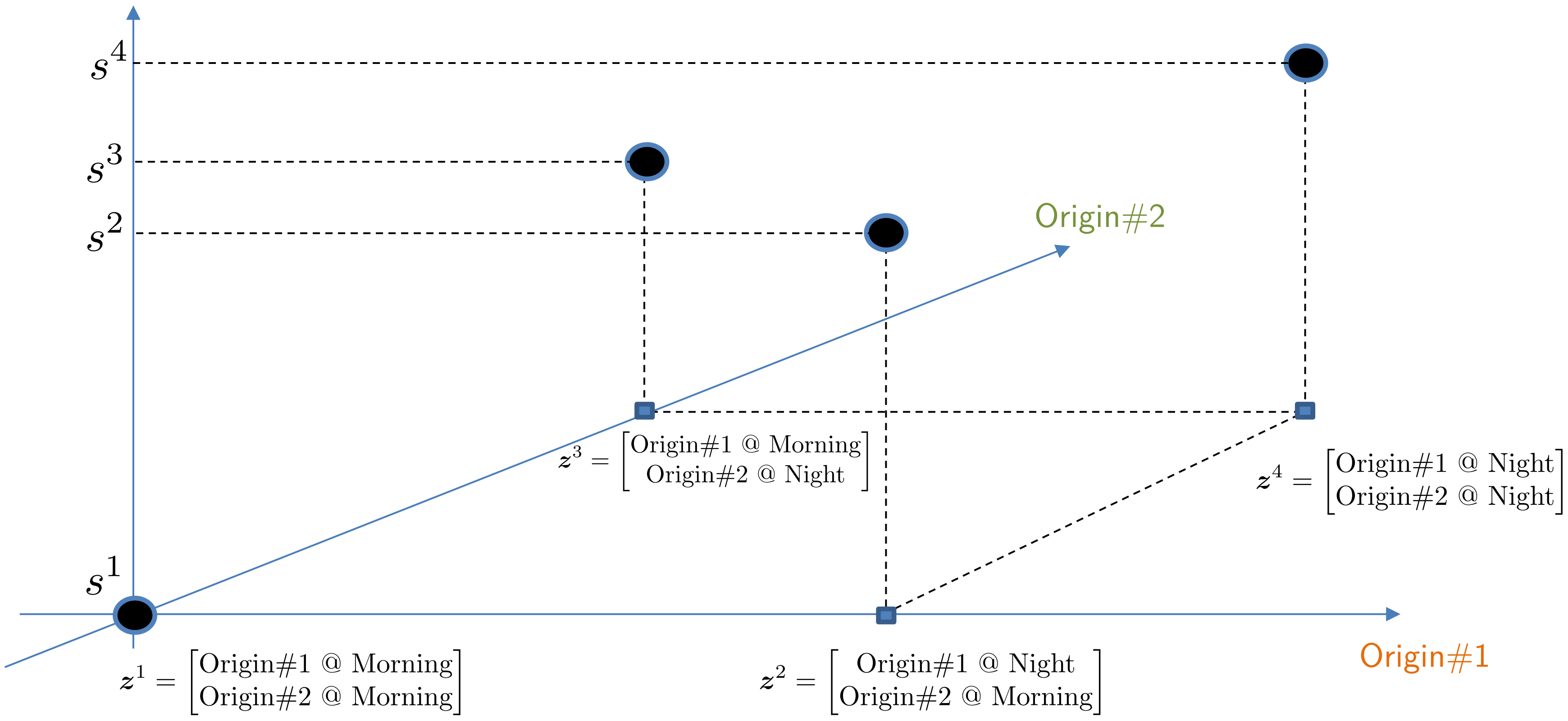

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$$\texttt{Unique}(\text{Origin\#1}) + \texttt{Unique}(\text{Origin\#2}) \geq \texttt{Unique}(\text{Origin\#1} \cup \text{Origin\#2})$$

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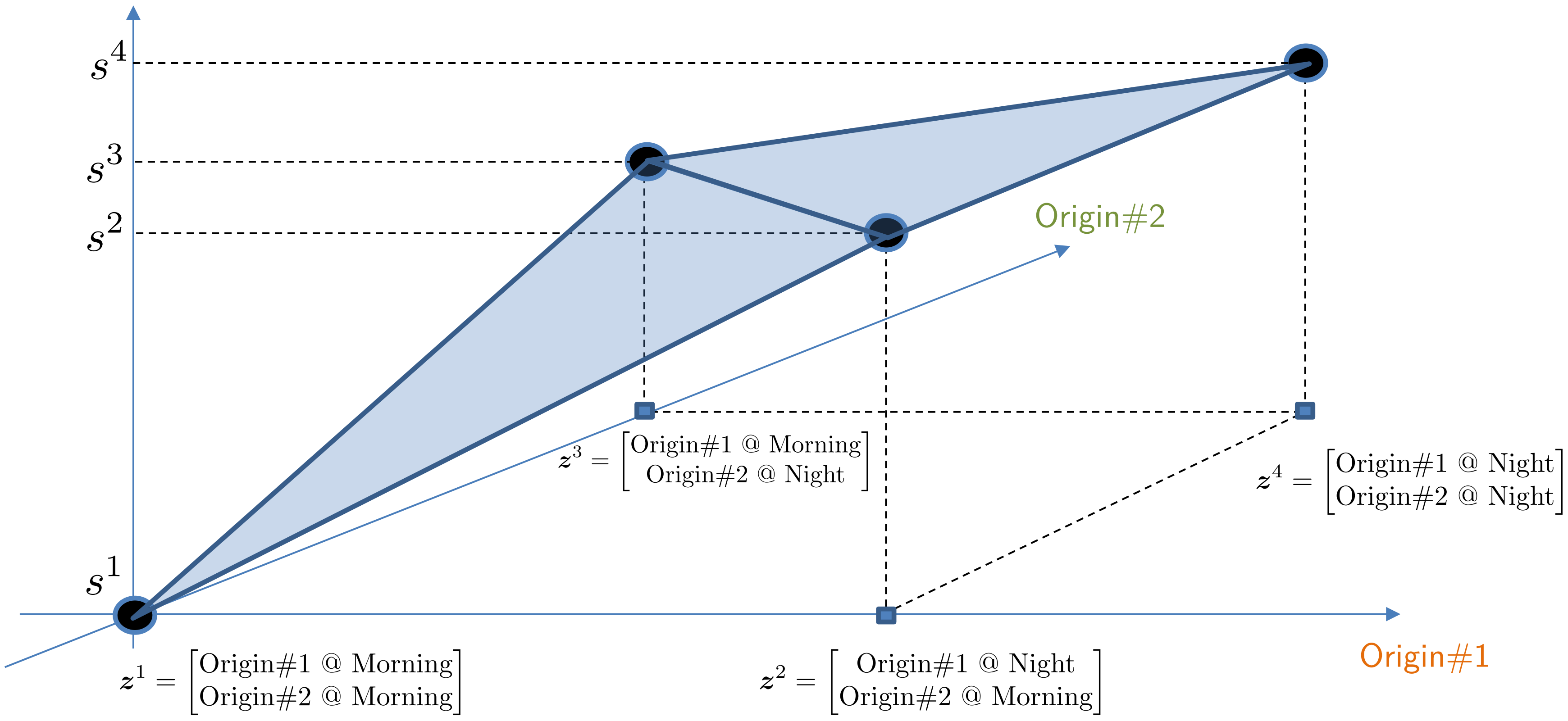

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Approximating the speed objective

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Step 1: Consider truck departure time combinations:

$$\boldsymbol{z}^1 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, \boldsymbol{z}^2 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, \boldsymbol{z}^3 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Night} \end{bmatrix}, \boldsymbol{z}^4 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Night} \end{bmatrix}$$

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Approximating the speed objective

Step 1: Consider truck departure time combinations:

$$\boldsymbol{z}^1 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, \boldsymbol{z}^2 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Morning} \end{bmatrix}, \boldsymbol{z}^3 = \begin{bmatrix} \text{Origin\#1 @ Morning} \\ \text{Origin\#2 @ Night} \end{bmatrix}, \boldsymbol{z}^4 = \begin{bmatrix} \text{Origin\#1 @ Night} \\ \text{Origin\#2 @ Night} \end{bmatrix}$$

Step 2: Compute speed for each combination:

$$s^i = \texttt{Speed}(\boldsymbol{z}^i)$$

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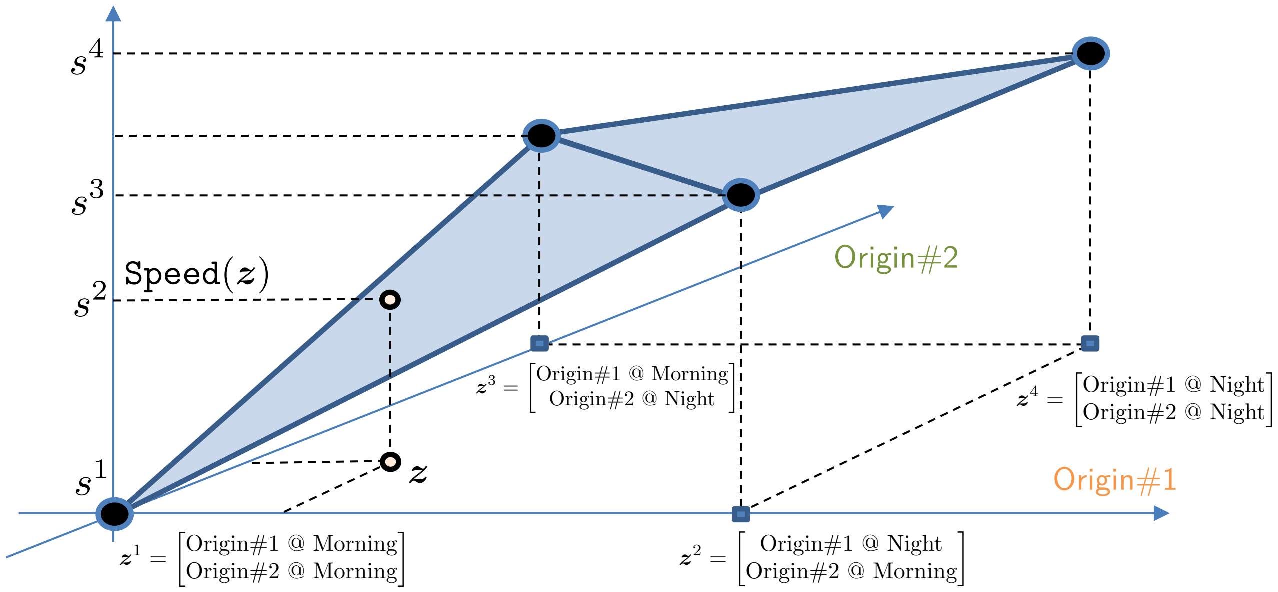

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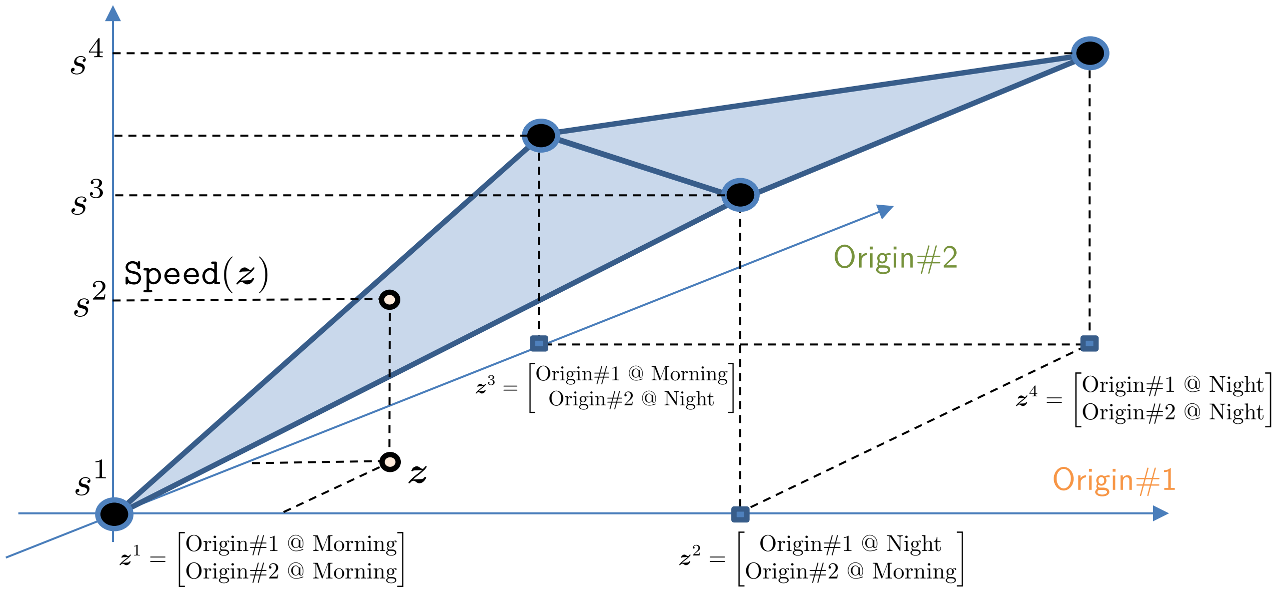
$$\texttt{Speed}(\boldsymbol{z}) = \max_{\boldsymbol{\alpha} \geq 0} \quad \sum_i \alpha^i s^i$$

$$\text{subject to} \quad \sum_i \alpha^i \boldsymbol{z}^i = \boldsymbol{z}$$

$$\sum_i \alpha_d^i = 1$$



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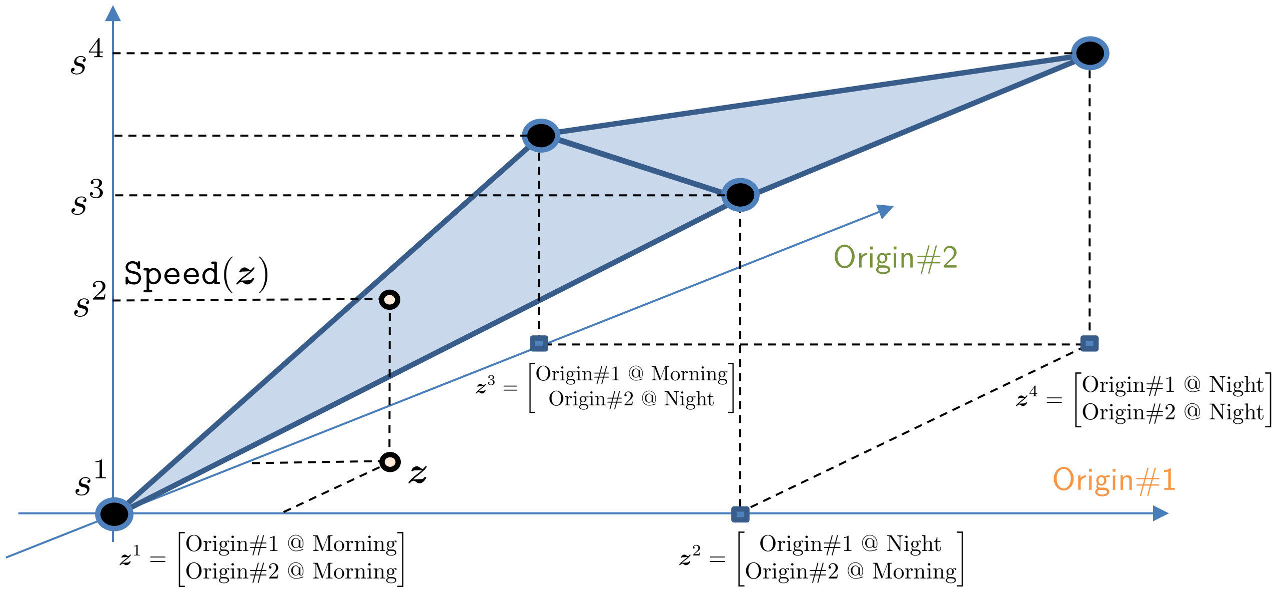
Parameter defining the optimal value of the LP



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Approximating the speed objective

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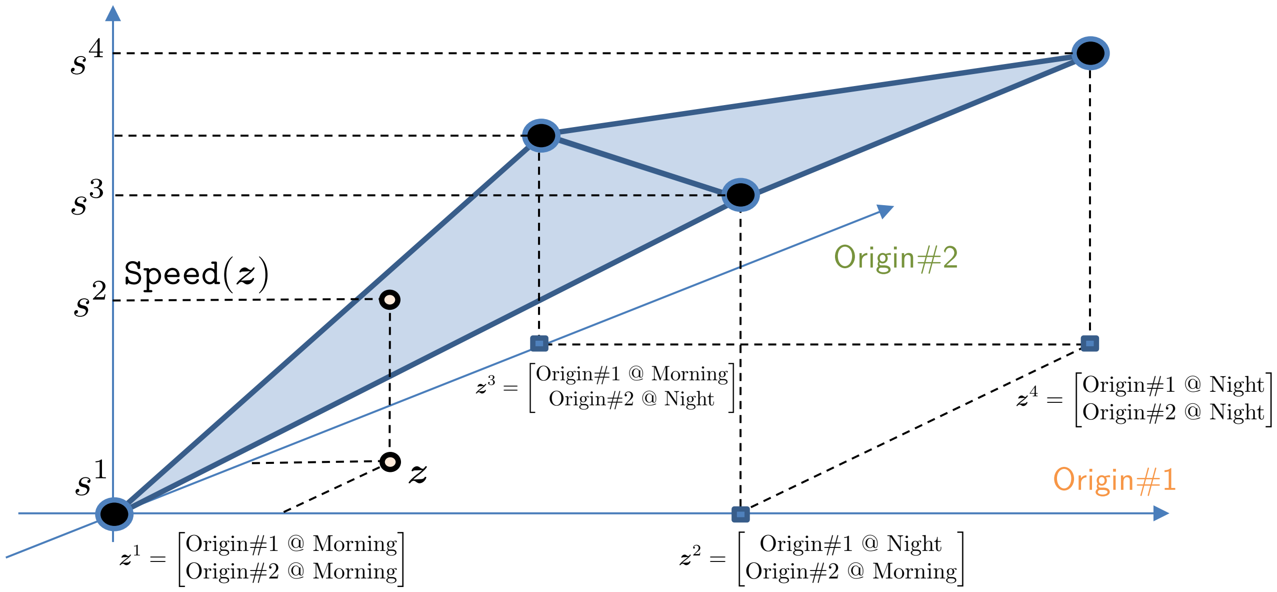

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Approximating the speed objective

$$
\begin{aligned}
\min_{\boldsymbol{p},\boldsymbol{y},\boldsymbol{z}} \quad & \texttt{NetworkCost}(\boldsymbol{p},\boldsymbol{y}) - \texttt{Speed}(\boldsymbol{z}) \\
\text{s.t.} \quad & (\boldsymbol{p},\boldsymbol{y}) \in \texttt{FeasibleNetwork} \\
& \boldsymbol{z} \in \texttt{FeasibleSchedule}(\boldsymbol{p})
\end{aligned}
$$

Step 3: Interpolation via parametric optimization:

$$
\begin{aligned}
\texttt{Speed}(\boldsymbol{z}) = \max_{\boldsymbol{\alpha}\geq 0} \quad & \sum_i \alpha^i s^i \\
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& \sum_i \alpha^i_d = 1
\end{aligned}
$$

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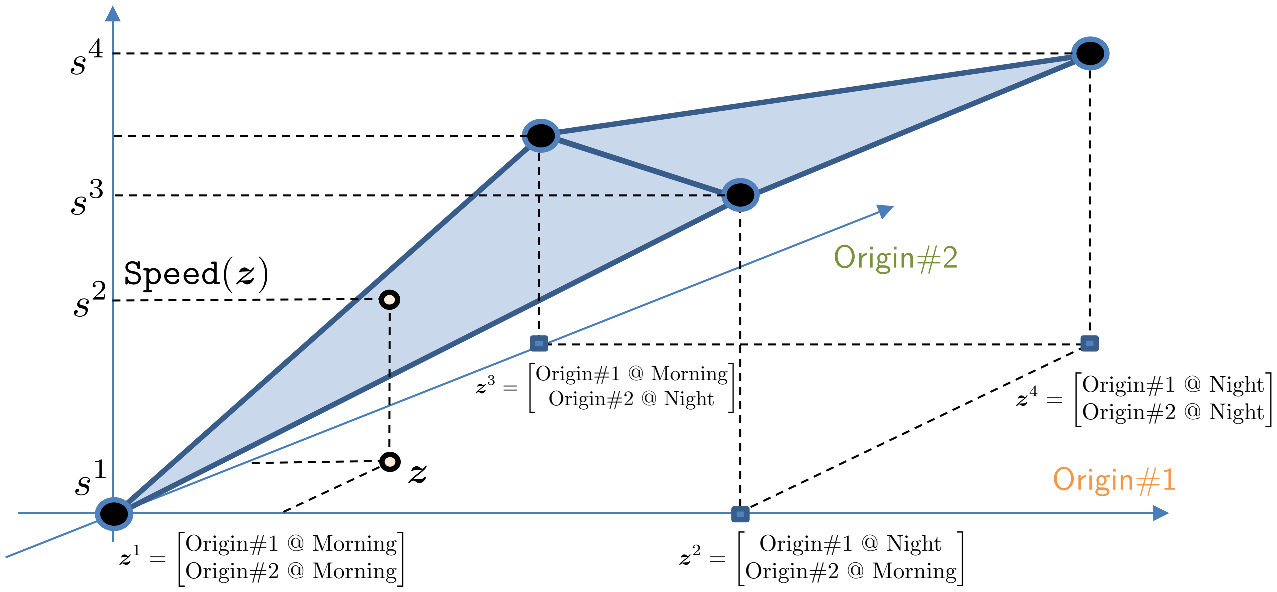

Approximating the speed objective

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{y},\boldsymbol{z},\boldsymbol{\alpha}>0} \quad & \texttt{NetworkCost}(\boldsymbol{p},\boldsymbol{y}) - \textstyle\sum_i \alpha^i s^i \\ \text{s.t.} \quad & (\boldsymbol{p},\boldsymbol{y}) \in \texttt{FeasibleNetwork}, \\ & \boldsymbol{z} \in \texttt{FeasibleSchedule}(\boldsymbol{p}) \\ & \textstyle\sum_i \alpha^i \boldsymbol{z}^i = \boldsymbol{z}, \textstyle\sum_i \alpha^i = 1 \end{aligned}$$

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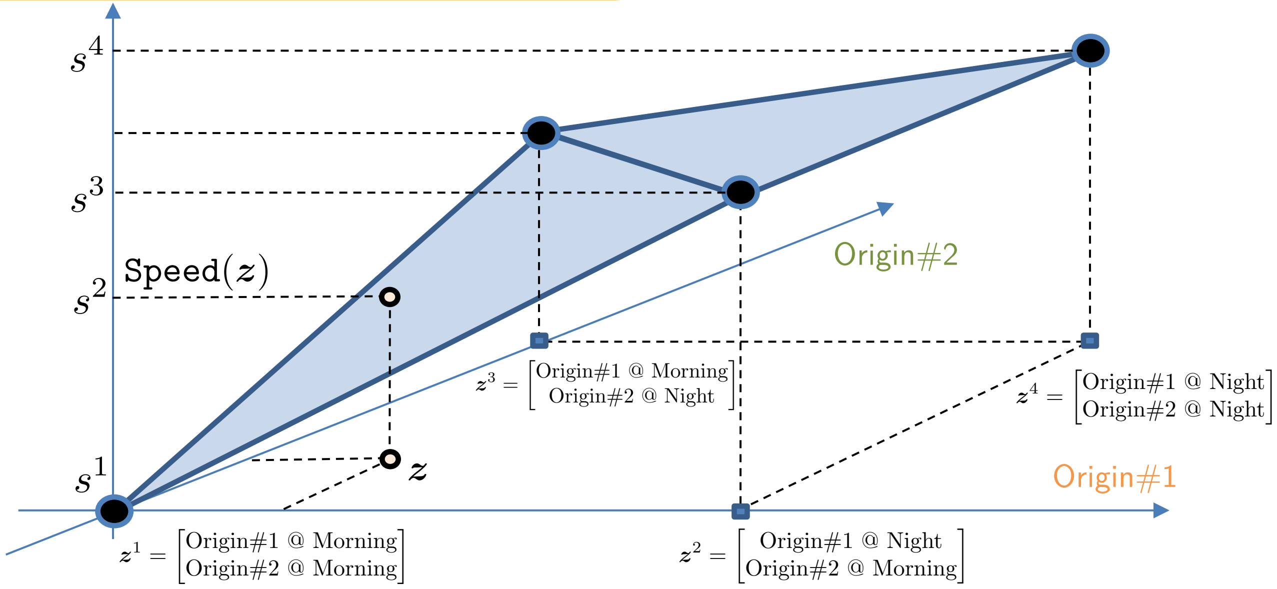

Outline

- Motivation
- Problem formulation
- Solution strategy
- Results

Results on a randomly generated datasets

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Results on 4 randomly generated datasets

- Location of nodes generated at random (cost of operating a truck proportional to distance)
- Items stored in each warehouse randomly generated
- Assuming a 0.1 conversion factor from speed and cost

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Speed parametric approximation compute by subsampling datapoint

Algorithm 1: Unique Items Approximation

```
Inputs: κ, D, F;
for d ∈ D do
    Initialize Λ_d = ∅;
    Sort by number of unique item the FCs that can offer 1-day delivery to DS d;
    Compute the set Z_d of all possible combinations of speed lane assignments for top κ FCs;
    For all z_d^i ∈ Z_d, add the tuple (z_d^i, U_d(z_d^i)) to the set Λ_d;
    Compute the speed lane assignments from (12) for remaining (n_f − κ) FCs;
    For all z̃_d^i ∈ Z̃_d, add the tuple (z̃_d^i, U_d(z̃_d^i)) to the set Λ_d;
Return: Set of vectors and cost coefficients Λ_d for d ∈ D.
```

Take all combination for top κ origins

Results on a randomly generated datasets

Baseline method only considering cost

n_F	n_D	Costs	App. Rev.	Rev.	Cost - (App. Rev.)	Cost - Rev.	κ	%Gap
10	10	689.1	-	342.8	689.1	346.3	-	0
10	10	698.5	383.7	383.7	314.8	**314.8**	1	0
10	10	698.5	383.7	383.7	314.8	**314.8**	5	0
10	10	698.5	383.7	383.7	314.8	**314.8**	10	0
10	20	1419.2	-	370.7	1419.2	677.7	-	1.8
10	20	1459.7	405.9	407	647.9	645.7	1	3.4
10	20	1434.1	397.8	400.7	638.5	632.7	5	2.8
10	20	1435.5	403.5	403.5	628.4	628.4	10	2.5
20	10	1392.2	-	770.4	1392.2	621.8	-	0.1
20	10	1422.9	807.3	815.7	615.6	607.2	1	0.1
20	10	1422.9	807.3	815.7	615.6	607.2	5	0.1
20	10	1413.6	806.5	811.2	607.1	**602.4**	10	0.1
50	10	3375.8	-	1845.2	3375.8	1530.6	-	1.8
50	10	3528.2	2010	2032.2	1518.2	1496.0	1	7.5
50	10	3499.0	2031.5	2039.1	1467.5	1459.9	5	3.8
50	10	3489.8	2025.3	2037.3	1464.4	**1452.4**	10	3.6
100	100	62204.2	-	3651.1	62204.2	25693.2	-	2.8
100	100	65911.4	4159.3	4175.7	24317.9	24154.2	1	11.2
100	100	65531.7	4156.8	4173.1	23963.2	23800.5	5	9.5
100	100	65258.6	4154.3	4172.1	23714.9	**23537.2**	10	8.4

8.36% improvement over baseline

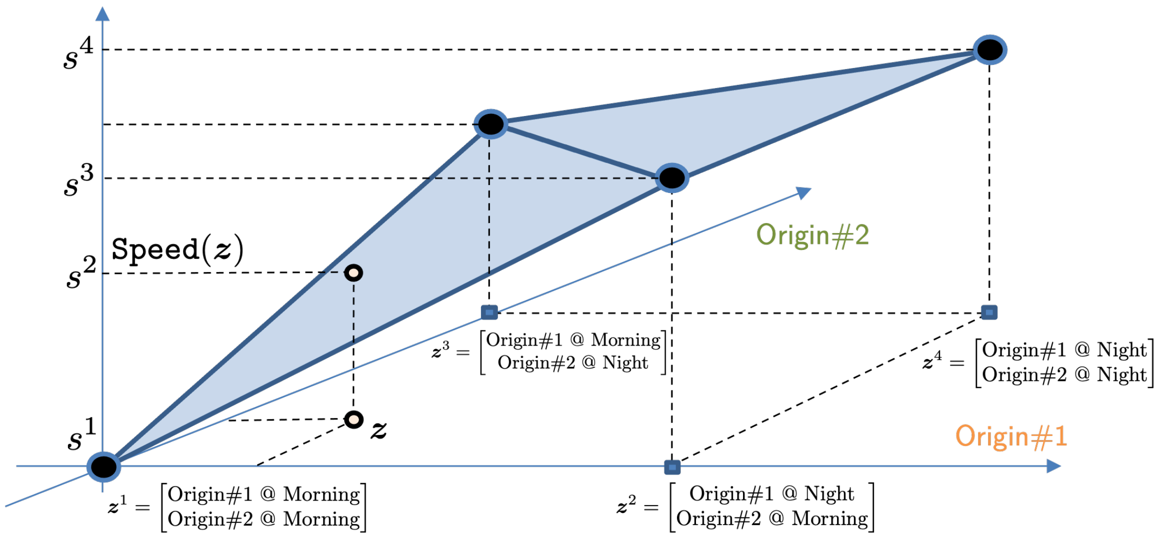
Summary

Problem

- Speed and cost should be jointly optimised
- Speed objective is submodular and can be evaluated with a data query

Solution

- Leverage parametric optimization
- Subsampling strategy to reduce complexity



Results

- 8.36% speed and cost benefits compared to baseline
- No additional computational cost compared to baseline